LEARNING ADVANCED MATHEMATICS BY ACOUSTICS – YES WE CAN!

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ABSTRACT

Experiences of transforming the context of the subject matter of an advanced, graduate mathematics course with overly abstract and specific concepts into integrated, real world acoustic context are discussed while still keeping the most important learning objectives of the original advanced mathematical course. The typical student of the course is interested in acoustics, has theoretical and experimental experience of acoustics but is frequently neither skillful nor interested in abstract, advanced mathematics. The only prerequisites are completed courses in mathematics and mechanics at undergraduate level. The learning/teaching spiral of the course typically starts by introducing a concrete example, such as a baby on a swing, and then mathematically trying to model the baby's motion by the well-known Newton´s Second Law. Then the equations of motion are slightly generalized and a general solution is derived. Subsequently, home assignments are handed out where the method taught is applied to a slightly new acoustical situation. The course material thus relates to the students' personal experience and to prior courses, while amplifying the transfer value to new applications, with increasing learning motivation and attention. The outcome of the home assignment frequently shows new insights, such as amplitude dependent period time, a harmonic excitation resulting in multi-harmonic response and other uncommon linear acoustical results. The results are due to the non-linearity of the system analyzed and are normally uncommon from previous modelling point of view but are, nevertheless, very common from practical point of view. Almost all students have experienced the results from real world practices although have never modelled it! Therefore, it is easily for the student to both assimilate the novel knowledge and accommodate it. Then the learning/teaching spiral includes continuously more realistic modelling features, such as damping, friction and viscosity. New insights are subsequently drawn. The course are learned by consecutively shifting between the concrete (relevant acoustical examples, phenomena, applications, hands-on, practical problem solving) and the abstract matter (symbols, principles, fundamental understanding). The students practice problem solving, evaluation and critical thinking skills. However, and most surprising, the students have learned an abstract perturbation method to solve non-linear, ordinary and partial differential equations – an advanced, graduate mathematical solution method, by transforming the overly abstract and specific mathematical context into an integrated, real world and well experienced acoustic context. And yes – we can learn advanced mathematics by acoustics!

KEYWORDS
Mathematics, Acoustics, Non-linear differential equation, Abstract versus concrete, CDIO Implementation, Standards: 1, 2, 3, 4, 5, 6, 7, 8, 11

INTRODUCTION

In this paper an innovative engineering method to teach advanced mathematics is presented and discussed. Experiences of transforming the context of the subject matter of an advanced, graduate mathematics course with overly abstract and specific concepts into integrated, real world acoustic context [SD2180 - Non-linear Acoustics 6.0 ECTS credits] are discussed while still keeping the most important learning objectives of the original advanced mathematical course. A typical student of the course is interested in acoustics, has theoretical and experimental experience of acoustics but is frequently neither skillful nor interested in abstract, advanced mathematics. The only prerequisites are completed courses in mathematics and mechanics at undergraduate level.

COURSE INFORMATION

The course is intended for a first year master student and is learning-centered, supporting a view of learners as active participants in their own learning while using continuous formative assessments with no need for a final examination. Perturbation methods learned include straightforward expansion, Lindstedt-Poincaré method, method of multiple scales, method of harmonic balance, method of averaging and basic numerical methods. The course spans over almost one semester, consists of ten to twelve two hours lessons and ending with a seminar given by each student individually. The typical number of students is between five and fifteen. The learning outcomes are continuously assessed by totally about four to five home assignments where methods learned and skills developed during the course are applied to new situations while requiring both analyzing and evaluation of the results and methods used. It is permissible to cooperate on the assignments, but they must be handed in individually and written in pencil. A recently published scientific paper using some of the method learned is individually reviewed during the final part of the course. The method used and results shown are critically evaluated while also suggesting some alternative approaches. The results of this review are given at a seminar and in a short individual report, covering approximately one to two A4 pages. The course has evolved during the last ten years into this innovative format.

COURSE START

The course starts by introducing a few concrete examples, such as a baby on a swing, see Figure 1, and a heavy engine standing on a rubber mount, see Figure 2. The two examples

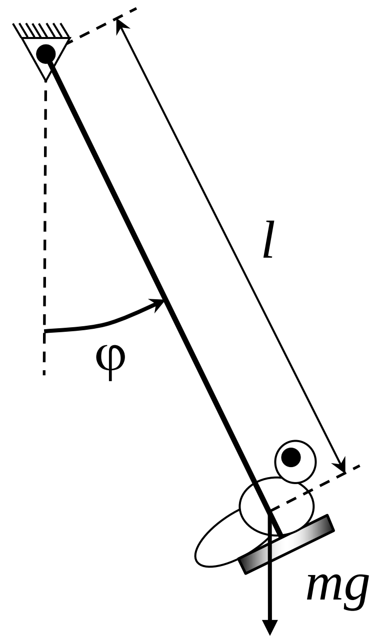

Figure 1. Baby on a swing

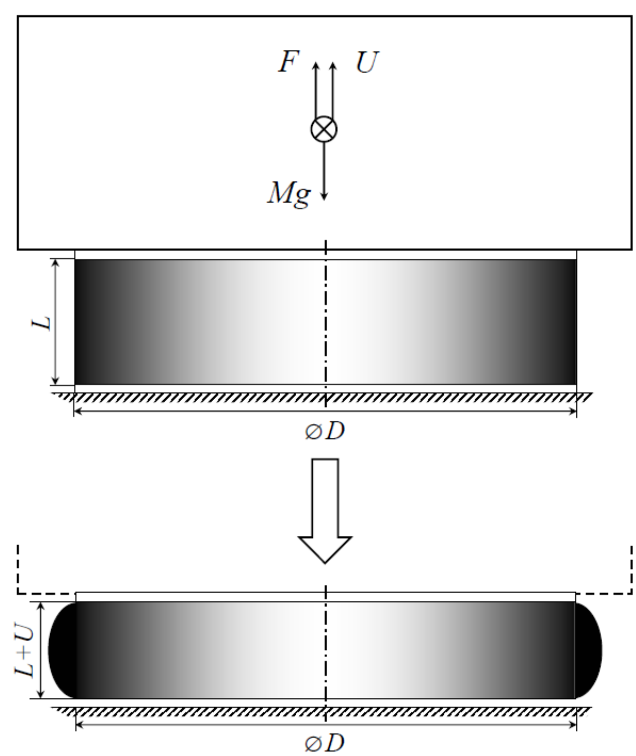

Figure 2. Heavy engine standing on a rubber mount

relate to the students' personal experience. In particular, the baby on a swing is a well-known experience for almost all students and the heavy engine standing on a rubber mount has frequently been experienced by many engineering students – at least they have seen it in cars. The course continues by the students mathematically trying to model the baby's and engines' motion by the well-known Newton´s Second Law. Finally, the derived equations of motion is inspected shallowly while trying to identify the source of non-linearity without trying to solve them. The main reason is to learn that simple, common and real world vibrating systems frequently embody non-linearities.

NEXT LESSONS

The equations of motion derived in the first lesson are slightly generalized and a general solution is derived. Subsequently, home assignments are handed out where the method taught is applied to a slightly new acoustical situation. The course material thus relates to the students' personal experience and to prior courses, while amplifying the transfer value to new applications, with increasing learning motivation and attention. The outcome of the home assignment frequently shows new insights, such as amplitude dependent period time, a harmonic excitation resulting in multi-harmonic response and other uncommon linear acoustical results. The results are due to the non-linearity of the system analyzed and are normally uncommon from previous modelling point of view but are, nevertheless, very common from practical point of view. Almost all students have experienced the results from real world practices although have never modelled it!

SUBSEQUENT LESSONS

In the subsequent lessons the learning/teaching spiral includes continuously more realistic modelling features, such as damping by viscosity and friction, and forced systems, see Figures 3 to 5. Clearly, novel insights are subsequently drawn. The course are learned by

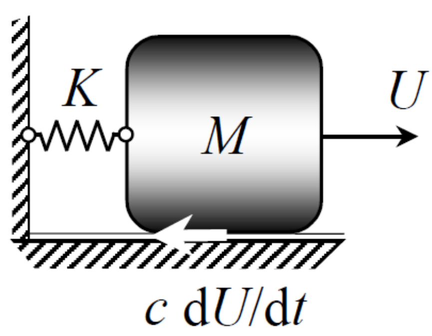

Figure 3. Viscous damping of a vibrating engine

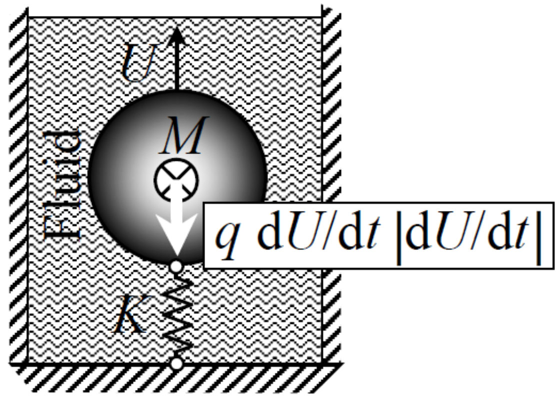

Figure 4. Non-linear damping of an immersed vibrating body

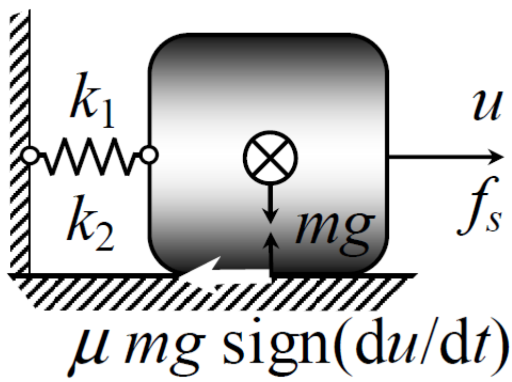

Figure 5. Forced vibration of a friction damped vibrating engine

consecutively shifting between the concrete (relevant acoustical examples, phenomena, applications, hands-on, practical problem solving) and the abstract matter (symbols, principles, fundamental understanding). The students practice problem solving, evaluation and critical thinking skills. However, and most surprising, the students have learned an abstract perturbation method to solve non-linear, ordinary and partial differential equations – an advanced, graduate mathematical solution method, by transforming the overly abstract and specific mathematical context into an integrated, real world and well experienced acoustic context.

COURSE EVALUATION

The course has been evaluated both individually in writing and together in a group discussion with the teacher. Regardless of method used, the evaluation shows that the innovative course structure is appreciated by the students; they are proud to be able to analytically solve advanced, non-linear differential equations while modelling phenomena normally uncommon from previous modelling point of view but are, nevertheless, very common from practical point of view. Moreover, they prefer home assignments instead of a final examination mainly due to complexity of the problems, requiring a day or so to solve a home assignment where the limited time at a written final examination would easily increase the possibilities to make errors or, even worse, totally block the student from finding a solution to the problem. Additionally, they point out that solving the home assignments in their own pace, possibly with help of others, reflects better the working practice of an engineer compared to writing a final examination individually under a time limitation. Furthermore, they are surprised to be able to satisfactorily understand recently published scientific papers on non-linear acoustics containing advanced, non-linear differential equations. The grades of the students from the course are typically very high. In the latest class all students received an A – the highest grade possible. A typical improvement suggestion from course evaluations includes time adjustments for hand-in dates for home assignments. The latest course evaluation suggested even more examples of acoustical non-linearities – possibly using some Youtube clips.

DISCUSSION

The prevailing constructivist view of learning/teaching, both in a cognitive (Piaget, 1972) and social (Vygotsky, 1987) setting; having its origin in the epistemological questions Immanuel Kant put forward on the outer and inner world, and on their interrelation. That is, how we perceive the outer events and objects; resulting in his celebrated distinction between *noumenon* and *phaenoenon*. Inspired by David Hume's criticism on rationalism he concluded that knowledge is not equal to an inner rational reasoning independent on sensory impressions. Nor is knowledge, according to the constructivism, equal to the idea of empiricism as solely sensory impressions. Instead, knowledge is a process where a person constructs inner cognitive structures through interactions with the outer world; by inner rational reasoning interacting with those of sensory impressions. The interpretation of sensory impressions from the outer world could be in a broad sense; ranging from reception of external information, observable through the usual senses, to reception of internal information, arising introspectively; thus, encompassing both sensing and intuitive learning styles (Felder & Silver, 1988). The common example shown of a mother saying 'a ball' to her baby while pointing at a filled circle in book is an illustrative incident of cognitive constructive learning: Before, 'ball' was identical to the child's own playing ball; now the baby assimilates

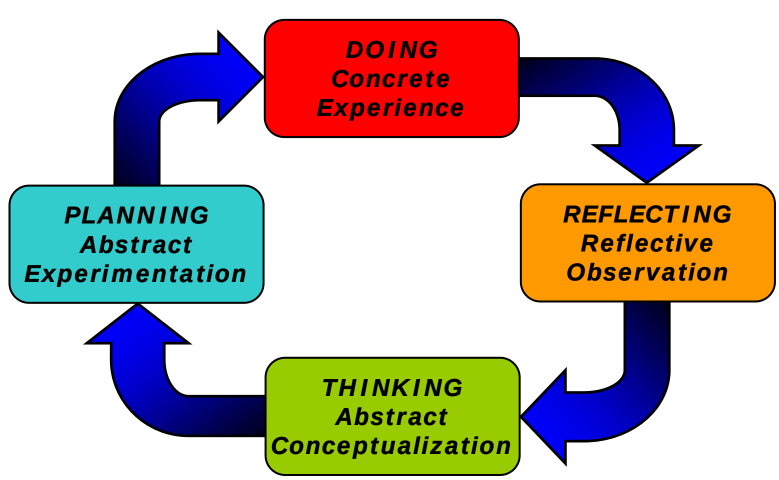

Figure 6. Kolb learning spiral (Kolb, 1966)

(Piaget, 1972) an empirical sensory impression of the filled circle to the existing inner cognitive structures of his own ball, thereby adjusting his definition of a 'ball' by accommodation (Piaget, 1972) to the surroundings. With respect to the innovative course discussed in this paper, almost all students have experienced the results from real world practices of the examples shown in the course although they have never modelled it. Consequently, compared with traditional teaching, this approach bridges the gap between theory and reality, making assimilation and accommodation easier. Subsequently, the learning/teaching spiral includes continuously more realistic modelling features and new insights are subsequently drawn. The Kolb's learning spiral (Kolb, 1966) in Figure 6, highlights very well the students' excellent learning progression in the course: The starting point is the first concrete learning experience, the second step involves observations and reflections on that experience, the third includes using abstract concepts and generalizations to make sense of the reflections while the forth step leads on to testing the implications derived from the abstractions in new learning situations.

Based on extensive engineering education experience and educational psychology, Felder & Silverman (1988) proposed five constructivist learning and teaching sub-style dimensions. With respect to the innovative course presented in this paper, there is a strive to motivate learning maximally, achieved by *e.g.* relating course material to the students' personal experience and the course to prior and ensuing courses, thus addressing all learners and particularly those with *global* and *inductive* learning styles. We seek to employ the scientific process as a metaphor to teaching/learning both in the course (Hult, 1998). First the big picture is presented [*global*] as well as concrete physical phenomena [*sensing*/*inductive*], then an analytical/numerical model is derived [*intuitive*/ *deductive*/*sequential*] and other phenomena are deduced including possible model limitations [*deductive*/*sequential*] and, finally, some promising new applications are presented [*sensing*/*deductive*/*sequential*]. The learners are promoted to take active part in the lectures; through active discussions, questioning, arguing, experimentations, small group brain storming *etc* [*active*], but also through active reflective observations facilitated by using micro-pauses during the lectures [*reflective*]; being critical incidents of activities which facilitate knowledge construction rather than reception of information. The future goal is to increase these lecture activities by *e.g.* covering less subject matter "coverage is the worst enemy to learning", particularly through reducing the material already covered by text books, to motivate more, acquiring a deeper view, to pause more, while expanding the repertoire by applying contemporary lecturing techniques that engage successful learning (de Winstanley & Björk, 2002). We have found that occasionally finishing a lecture with an open-ended, highly relevant question that calls for analysis [*reflective*] and synthesis [*reflective*/*global*] improves learning sharply; the main reason being the increased learning time and number of learning events, while reflecting over the question wherever and whenever after the lecture. This spacing effect and the non-trivial, highly relevant and to some extent complex question promotes long-term retention and amplifies the transfer value to new applications, while increasing learning motivation and attention during the following lecture. We have also found that highly complex theories, normally required for 'real world' problems, are best learned by consecutively shifting between the concrete (relevant examples, phenomena, applications, hands-on, practical problem solving) [*sensing*/*active*] and the abstract matter (symbols, principles, fundamental understanding) [*intuitive*/*reflective*].

Assessments that reflect the instructional objectives engender appropriate learning activities while also motivating students, offering timely and informative feedback on their work, assisting them in internalizing the standards and quality philosophies of the discipline and finally, assuring auditable course quality (Gibbs, 1994). To achieve this, the disrupting summative assessments providing untimely feedback (read: too late) are replaced by continuous formative assessments to a much greater extent (Hunt & Pellegrino, 2002). We envision a synthetic design process for assessments, where the data stream generated by learning activities are smoothly and continuously assessed; thus making summative assessments superfluous. In fact, the home assignments in the course discussed in this paper are already playing the ultimate role of integrated learning activities meeting the instructional objectives while formatively and continuously assessing the students, where methods learned and skills developed during the course are applied to new situations. The cognitive taxonomy levels aimed at in that course span over a wide spectrum; from application to the highest, evaluation, according to Bloom (1972). The assignments are promptly corrected and commented for optimal feedback. The instructional objectives corresponding to the highest cognitive levels and to those within the affective domain are mainly assessed and learned during a group-wise scientific paper review process, as these imponderables are suitably learned by a social activity later to be internalized in a cognitive process of the individual (Vygotsky, 1987). Subsequently, the higher level acquired thinking skills and attitudes are demonstrated in a report and at a seminar. We are particularly satisfied to be able to explore a valid assessment method and suitable student learning activities for those higher-level instructional objectives, as achieved high-level thinking skills and attitudes eventually form a profession—in this course constituting ‘the mathematical skillful engineering acoustical researcher’.

CONCLUSIONS

To analytically solve non-linear differential equations is generally a complex issue, typically learned at an advanced, graduate mathematics course. However, in an innovative course design presented in this paper, the subject matter is transformed into an integrated, real world acoustic context while applying and simultaneously learning perturbation methods including straightforward expansion, Lindstedt-Poincaré method, method of multiple scales, method of harmonic balance and method of averaging. The acoustical examples applied in the course relate to the students’ personal experience showing amplitude dependent period time, a harmonic excitation resulting in multi-harmonic response and other uncommon linear acoustical results. The results are due to the non-linearity of the system analyzed and are normally uncommon from previous modelling point of view but are, nevertheless, very common from practical point of view. Almost all students have experienced the results from real world practices although have never modelled it! Student evaluations show that the innovative course structure is appreciated by the students and that they learn advanced mathematical solution methods in a new way.

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BIOGRAPHICAL INFORMATION

Leif Kari is a Professor in Engineering Acoustics and Head of School of Engineering Sciences at KTH Royal Institute of Technology, Stockholm, Sweden. His current research focuses on structure-borne sound and smart materials and is interested in educational management. He has a long experience as director of undergraduate education, director of a master program in Sound and Vibration and director of a five-year program in Engineering Physics at KTH. He is frequently involved in analyzing organizational structures, including an ongoing review of the management and operational support at KTH.

Anna-Karin Högfeldt is a Lecturer and Director of Faculty Training at KTH Royal Institute of Technology, Stockholm, Sweden. She is actively involved in Nordic and International/cross-continental education evaluation, development and research projects. She is one of the main authors of the book Guide to Challenge Driven Education, which originates from a collaboration project with partners in East Africa. At KTH, she has worked ten years strategically to support management, schools, education program directors and individual teachers to strengthen education and system level approaches.

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