# Integrated Learning for Engineering and Mathematics Modules

**Fan Kee Seng**

Mathematics & Science Department, Singapore Polytechnic

**Yoong Yuen Soo, Nashatar Singh, Ng Geok Ling**

School of Architecture and the Built Environment, Singapore Polytechnic

## ABSTRACT

The Department of Mathematics and Science (MS) and the School of Architecture and the Built Environment (ABE) in Singapore Polytechnic (SP) have adopted the CDIO educational framework in the Diploma in Civil Engineering and Management (DCEM) course. To meet CDIO standards, both MS and ABE have actively worked to integrate the learning of engineering and mathematics modules through the use of a common Engineering Formulae booklet.

The paper describes (a) the systematic approach which MS and ABE have taken, (b) the results of student perception feedback and (c) the recommendations for further refinements and improvements.

## KEYWORDS

CDIO, mathematics, integrated learning, linkage, engineering, formulae booklet, reference module

## INTRODUCTION

In this paper we focus on the CDIO Standard 3 — Integrated Curriculum [1]. To meet this standard, the curriculum must be designed with mutually supporting disciplinary modules, with an explicit plan to integrate them. Disciplinary modules make explicit connections among related and supporting content and learning outcomes. The course management team members play an important role in designing the integrated curriculum by suggesting appropriate disciplinary linkages, as well as opportunities to address specific CDIO learning outcomes in their respective teaching areas.

## METHODOLOGY

MS and ABE formed a team to look into how engineering and mathematics modules could be integrated so as to enable students to relate better on how mathematics is applied to solve engineering problems. Module coordinators of engineering modules first compiled a list of formulae that are needed in the teaching and learning of their modules.

A total of 206 formulae from 16 engineering modules were gathered and compiled into a common Engineering Formulae booklet. In the booklet, each formula is indicated with a reference to the mathematics module that the students will be learning. The reference shows the mathematics module code and the topic of the mathematics module notes where the details of the formula are being taught.

Similarly, in each mathematics module, references are also indicated. Each reference shows the formula and the page number of the Engineering Formulae booklet where the mathematics is applied to engineering modules. Hence, there are cross references of engineering and mathematic modules for integrated learning [2]. The Formulae booklet is given to all first year students. The students will use the Formulae booklet throughout the three-year DCEM course.

## RATIONALE

In most engineering modules, formulae are given to students to solve engineering problems. The concepts and principles of these formulae are not explained to students as these involve enormous amounts of mathematical steps and manipulations. Hence, students do not have a good understanding of the formulae they have used. To bridge this gap, it is necessary to link these formulae to the relevant mathematics topics where the concepts and principles will be explained to promote their understanding of the engineering content [3]. It is important that students have a good understanding of engineering content for the future advancement of their careers and for further studies in the engineering field.

## EXAMPLES ON LINKAGES FOR INTEGRATED LEARNING

To illustrate how the cross references of engineering and mathematic modules for integrated learning are carried out using the Formulae booklet, I shall use a simple example to explain my point. In this example, students are asked to find the area of a circle with a radius of 5 cm in an engineering module. Students will first obtain the formula from the Formula booklet which is given as $A = \pi r^2$. They will then substitute $r$ = 5 *cm* into the formula as shown below: $A = \pi(5)^2 = 78.54\,cm^2$.However, the students may not know how to solve the problem if they are asked to find the radius of the circle given that its area is $41\pi\ cm^2$. In the Formulae booklet, a formula like this is linked to the mathematics module where the manipulation of the formula is taught and explained. Hence students will have a better understanding and ability to solve engineering problems after learning the mathematics modules. (Please note that this problem seems easy as it is purely used for illustration purposes. Engineering formulae are usually much more complicated than this.)

Below is an example on how engineering formulae in the Formulae booklet are linked to the mathematics modules. Please take note of the explanation in the callout (in light blue) below.

Module Name: Structural Mechanics (Engineering Module)
Module Code: BE752Y
Module Coordinator: Rose Huang
Formulae used:

1. Direct stress $f = \dfrac{F}{A}$

2. Strain $\varepsilon = \dfrac{\Delta l}{l}$

Linkage to mathematics module is shown in the yellow box.

3. Hooke's law $E = \frac{f}{\varepsilon}$

4. Bending stress $f = \frac{M \cdot y}{I_{xx}}$ — **Integration (MS322Z, Chapter 4)**

5. Moment of inertia $I = I_{NA} + A \cdot h^2$ — **Integration (MS322Z, Chapter 3)**

6. Euler's buckling load $P_{cr} = \frac{\pi^2 EI}{l_e^{\ 2}}$ — **Differential Equations (MS322Z, Chapter 8)**

Below is an example on how the working in a mathematics module is linked to the formula in the Formulae booklet. Please take note of the explanations in the callouts (in light blue) below.

From the mathematics module MS322Z Chapter 8, students learn how to find the deflection equation and the maximum deflection of a beam of length L. The beam has a uniform cross-section and is freely supported at both ends. It carries a uniformly distributed load (UDL), including its own weight, of w per unit length shown in Figure 1.

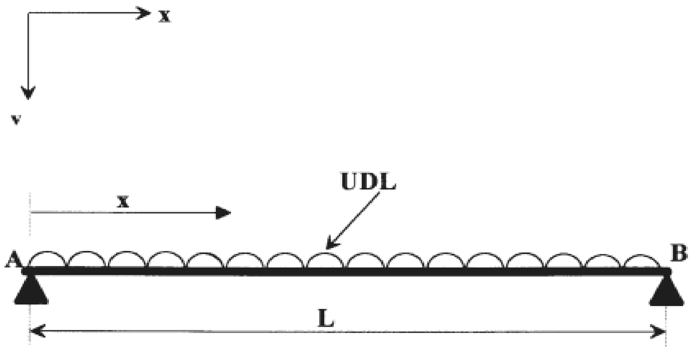
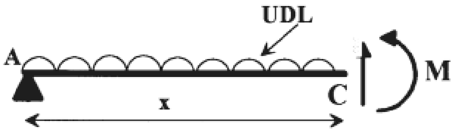



Figure 1. A Simply Supported Beam with Uniformly Distributed Load

Total load = wL

Reaction at each support = $\frac{wL}{2}$

Revision of engineering module (Structural Mechanics)




Revision of engineering module (Structural Mechanics)

To find M --- use the free body diagram above

Take moment about point C

$$M + (wx)(\frac{x}{2}) = \frac{wL}{2}(x)$$

$$M = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$\therefore \quad \frac{d^2v}{dx^2} = -\frac{M}{EI}$$

Application of Mathematical Modelling

$$\frac{d^2v}{dx^2} = -\frac{w}{2EI}(Lx - x^2) \text{ ------- it is of form } \frac{d^2y}{dx^2} = f(x) \qquad \text{Eqn (1)}$$

Integrate wrt x

Application of Integration

$$\frac{dv}{dx} = \frac{-w}{2EI}(\frac{Lx^2}{2} - \frac{x^3}{3} + A) \qquad \text{Eqn (2)}$$

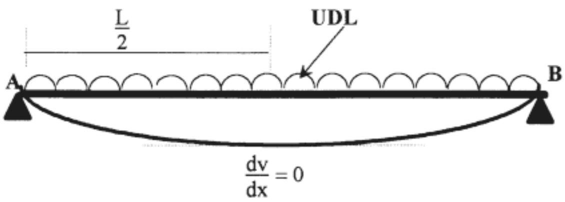
Integrate again wrt x

$$v = \frac{-w}{2EI}[\frac{Lx^3}{6} - \frac{x^4}{12} + Ax + B] \qquad \text{Eqn (3)}$$

To find the constant A, the boundary condition is:

when $x = \frac{L}{2}$ , $\frac{dv}{dx} = 0$, as shown below.

Boundary conditions as applied to the deflection at the mid-span of a simply support beam



That is, the gradient is zero when the deflection is maximum.

Application of Differentiation

Hence equation (2) $$\frac{dv}{dx} = \frac{-w}{2EI}\left(\frac{Lx^2}{2} - \frac{x^3}{3} + A\right)$$

becomes $$0 = -\frac{w}{2EI}\left[\frac{L}{2}\left(\frac{L}{2}\right)^2 - \frac{1}{3}\left(\frac{L}{2}\right)^3 + A\right]$$

$$0 = -\frac{w}{2EI}\left(\frac{L^3}{8} - \frac{L^3}{24} + A\right)$$

$$0 = -\frac{w}{2EI}\left(\frac{2L^3}{24} + A\right)$$, therefore $$A = -\frac{L^3}{12}$$

To find B, the boundary condition is:

when x = 0, v = 0 i.e. no deflection at the support.

Boundary conditions as applied to deflection at a support

Hence equation (3) $$v = \frac{-w}{2EI}\left[\frac{Lx^3}{6} - \frac{x^4}{12} + Ax + B\right]$$

becomes $$0 = -\frac{w}{2EI}(0 - 0 - 0 + B)$$, therefore B = 0

Hence the deflection equation is $$v = -\frac{w}{2EI}\left(\frac{Lx^3}{6} - \frac{x^4}{12} - \frac{L^3}{12}x\right)$$

Maximum deflection occurs at the mid span i.e. $x = \frac{L}{2}$

Hence, $$v = -\frac{w}{2EI}\left(\frac{Lx^3}{6} - \frac{x^4}{12} - \frac{L^3}{12}x\right)$$ becomes

$$v_{max} = -\frac{w}{2EI}\left(\frac{L^4}{48} - \frac{L^4}{192} - \frac{L^4}{24}\right)$$

$$v_{max} = -\frac{w}{2EI}\left(\frac{8L^4 - 2L^4 - 16L^4}{384}\right)$$

$$v_{max} = -\frac{w}{2EI}\left(-\frac{10L^4}{384}\right)$$

$$v_{max} = \frac{5wL^4}{384EI}$$

Linkage to the Formulae booklet Page 19, Equation No. 29 (b) of the engineering module: Steel Design & CAD

Refer to page 19 of Formulae Booklet, Equation No. 29 (b) (Steel Design & CAD)

Below is an example of how the contents in a mathematics module can be realigned to meet the needs of the engineering module.

In the mathematics module, students are taught how to solve for x in a quadratic equation of the form $ax^2 + bx + c = 0$. The equation can be solved by using factorisation or the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The derivation of the quadratic formula is also shown and explained to students. Students are then taught how to solve simple quadratic equations such as $2x^2 + 4x + 1 = 0$. However, some of the quadratic equations encountered in engineering modules could be more complicated. Hence in the mathematics module, it is necessary to give examples or exercises on how to solve more complicated quadratic equations. Below is an example of a difficult quadratic equation encountered in an engineering module (Concrete Design) covering singly reinforced rectangular section in bending.

Bending of the section will induce a resultant tensile force $F_{st}$ in the reinforcing steel, and a resultant compressive force in the concrete $F_{cc}$ which acts through the centroid of the effective area of concrete in compression shown in Figure 2.

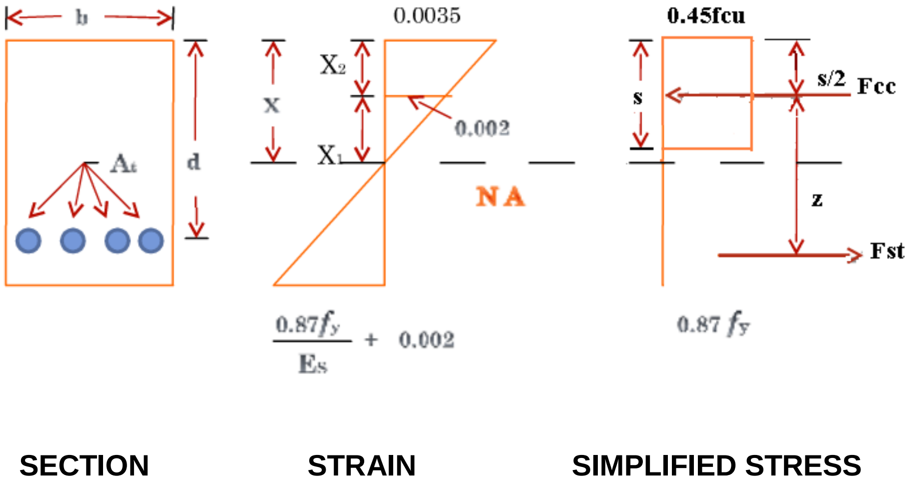



Figure 2. Cross-Section of a Beam and its Strain and Stress Diagrams

For equilibrium, the ultimate design moment, M, must be balanced by the moment of resistance of the section so that

$$M = F_{cc} \text{ x } z = F_{st} \text{ x } z \qquad \text{Eqn (4)}$$

where *z* is the lever arm between the resultant forces $F_{cc}$ and $F_{st}$.

$$F_{cc} = \text{stress x area of action}$$
$$= 0.45 f_{cu} \text{ x } bs$$

and

$$z = d - \frac{s}{2} \qquad \text{Eqn (5)}$$

So that substituting in Eqn (4)

$M = 0.45\, f_{cu}\, bs \times z$ and replacing s from Eqn (5) gives

$$M = 0.9\, f_{cu}\, b(d - z)\, z$$

Rearranging and substituting $K = \dfrac{M}{b d^2 f_{cu}}$

$$\left(\frac{z}{d}\right)^2 - \left(\frac{z}{d}\right) + \frac{k}{0.9} = 0$$

Solving this quadratic equation:

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right)$$

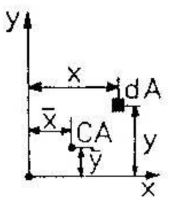
More complicated quadratic equation is encountered in this engineering module. Mathematics module needs to include more complicated equations as well.

which is the equation in the code of practice BS 8110 for the lever arm, z, of a singly reinforced concrete section. ………………………………………………… the remaining calculations are omitted to save space.

In addition, if engineering and mathematics modules are not well integrated or coordinated, some of the important basic concepts could be left out from the entire course. For examples, the first moment of area and the second moment area, as illustrated below, were found to be missing from the DCEM course.

First moment of area is defined as area multiplied by perpendicular distance. Referring to the plane in the Figure 3, the first moment of the element of area $dA$ about the $x$ - axis is $y\,dA$.

Therefore the first moment of the whole figure is $\int_A y\,dA$ about the $x$ - axis, the suffix $A$ indicating summation over the whole area. The first moment of the whole figure about the $y$ -axis is $\int_A x\,dA = \int_a^b x\,(y_u - y_l)\,dx$




Maths modules need to be realigned to include important basic concepts

Figure 3. An Element of Area $dA$

Second moment of area is defined as area multiplied by the square of perpendicular distance. If the first moment of the element $dA$ about an axis is multiplied again by the respective co-ordinate, we obtain the second moment of area, namely $y^2\,dA$ or $x^2\,dA$. The second moment of area of the whole figure: about the $x$ -axis $I_{xx} = \int_A y^2\,dA$ and about the $y$ -axis, $I_{yy} = \int_A x^2\,dA$. It is necessary that these two basic concepts are taught to the students in the mathematics modules and linked to engineering modules through the use of the Formulae booklet. However, sometimes it is not possible to keep adding content to existing mathematics modules. We can overcome this problem by removing non essential content from the mathematic modules. However, with the reduction of curriculum hours for mathematics modules, we may ultimately need to create a separate mathematics module called **reference module** for students to learn outside their curriculum hours. The reference module will encourage students to learn additional concepts or to acquire a better understanding of the application of mathematics to engineering modules. For example, they may learn the derivation of the mathematical model $\frac{d^2v}{dx^2} = -\frac{M}{EI}$ used for finding the deflection of beam mentioned in one of the examples above.

## SURVEY FINDINGS

A survey was carried out in Academic Year 2008/2009 Semester 2. Table 1 shows the responses to the five survey questions. The survey was conducted on 135 first, second and third year DCSE students. Each student was given a copy of the Formulae booklet.

- 90% of the students felt that the Formulae booklet helped them learn engineering modules better.
- 85% of the students felt that the booklet helped them understand or relate better how mathematical concepts are applied in engineering modules.
- 93% of the students felt that it was very convenient for them to refer to the booklet whenever they needed the formulae.
- 77% felt that, with the help of the booklet, they were motivated to learn engineering mathematics.
- 81% felt that they would use the booklet in their work or in their further studies after graduation.

Table 1
Responses of Student Survey

| No. | Survey Questions | Disagree or Strongly disagree | Neutral | Agree or Strongly agree | Average score (out of 5) |
|---|---|---|---|---|---|
| 1. | The Engineering Formulae booklet helps me learn engineering modules better. | 1% | 9% | 90% | 4.51 |
| 2. | The booklet helps me understand (relate) better how mathematical concepts are applied in engineering modules. | 3% | 12% | 85% | 4.36 |
| 3. | It is very convenient for me to refer to the booklet whenever I need the formulae. | 0% | 7% | 93% | 4.62 |
| 4. | With the help of the booklet, I am motivated to learn engineering mathematics. | 2% | 21% | 77% | 4.19 |
| 5. | I think I will use the booklet in my work or in my further studies after graduation. | 2% | 17% | 81% | 4.33 |

The findings based on general comments from these students are as follows:

- Many of the formulae for the modules on Geomatic and Structure Analysis have not been included in the Formulae booklet. They wondered why they still had to memorise formulae for these two modules.
- The Formulae booklet is particularly useful for weaker students.
- The compilation of the Formulae booklet is creative and a job well done.
- More details should be added in the Formulae booklet. The meaning of each term or symbol used in the booklet should be explained.
- Examples should be given for each formula.

## DISCUSSIONS

The survey findings obtained from students have been positive. The majority of the respondents felt that the use of a common Formulae booklet in the integrated teaching and learning of engineering and mathematics modules was useful and effective. This Formulae booklet is a useful tool to help the students assimilate and apply the mathematics knowledge they have learnt, in the engineering modules. Hopefully, students will be more motivated to learn what they deem to be useful for their course of studies [4].

One problem we encountered is that some formulae in the Formulae booklet could not be linked directly to the mathematics modules, as additional content is needed. Our future work for further integration includes the streamlining of mathematics modules in order to provide most of the content to meet the requirements of the engineering modules and also to expand or enhance the reference module. In this way, the content of the mathematics modules will be even more aligned with that of the engineering modules.

More importantly the feedback given by the students will also be incorporated into the Formulae booklet. In addition, the following are some possible improvements which the team can make to this Formulae booklet project:

- Include an introduction to explain the rationale and purpose of the Formulae booklet.

- Have clear and standardised format in all the mathematics notes informing students of the connection with the Formulae booklet. It can be a box with a suitable heading plus a link to the page of the Formulae booklet. This must be standardised for all the mathematics modules so that the students are "trained" to identify such boxes as links to their course of studies.

- Create an online version hosted on Blackboard with hyperlinks to the mathematics notes for easy reference for the students.

## CONCLUSION

The approach and specific innovation outlined in this paper conforms to the CDIO Standard 3, which advocates curriculum integration of disciplinary modules such as mathematics with engineering content and practices. From the survey findings in this pilot study, we found that majority of the students found this formulae booklet useful and effective. This gives the team the impetus to continue improving upon the Formulae booklet for the next few years.

## REFERENCES

**[1] Edward Crawley, Johan Malmqvist, Soren Ostlund and Doris Brodeur, "The CDIO Approach", Rethinking Engineering Education , 2007.**

**[2[ Roberts, Patricia L. and Kellough, Richard D. (2000), A Guide for Developing Interdisciplinary Thematic Units 2th edition, Columbus, Ohio: Merrill.**

**[3] Loepp, Franzie L. (1999), Models of Curriculum Integration. Journal of Technology Studies: Volume 25 (2).**

**[4] Czerniak, C. M., Weber W.B.J., Sandmann, A., Ahern J. (1999), A Literature Review of Science and Mathematics Integration, Science and Mathematics Integration: Volume 99 (8).**

## Biographical Information

Fan Kee Seng is a Section Head and a senior lecturer in the Department of Mathematics and Science (MS), Singapore Polytechnic (SP). He joined SP in 1982 and is a member of the Course Management Team representing MS Department. His interests are in educational pedagogy and technology, and engineering mathematics. He has worked as an engineer in consulting firms and government agencies on road, bridge, viaduct and building constructions.

Yoong Yuen Soo is a principal lecturer in the School of Architecture and the Built Environment, Singapore Polytechnic since 1981. His interests are in educational pedagogy for civil engineering technologists and environmental sustainability. He has worked in PUB as an engineer for water treatment works construction and water supply. He has a Bachelor's degree in Civil Engineering from the University of Singapore, a Master's of Science degree (Civil Engineering) from the National University of Singapore, and an Education Master's degree from Harvard University.

Nashatar Singh is a Section Head and senior lecturer in the Division of Civil Engineering of the School of Architecture & the Built Environment. He is a member of the Course Management Team responsible for course evaluation and assessment.

Ng Geok Ling is a senior lecturer for the Division of Civil Engineering in the School of Architecture and the Built Environment, Singapore Polytechnic since 1996. Her teaching focuses on CAD, civil engineering, environmental and water technology. Her current interests and research are in CAD education into BIM paradigm, promoting active learning in classroom teaching and student learning assessment methods. She graduated from the National University of Singapore with a Bachelor's degree in Civil Engineering and a Master's degree of Technology in Knowledge Engineering.

***Corresponding author***
Mr Fan Kee Seng
Singapore Polytechnic
500 Dover Road,
Singapore 139651
67721751
fankeeseng@sp.edu.sg

APPENDIX A

**Survey Questionnaire on the Use of Formulae booklet.**

Name: ___________________ (Optional)

Class: DCSE _______

(a) Please tick the appropriate boxes.
(b) 1 being strongly disagree and 5 being strongly agree

Strongly Disagree **to** Strongly Agree

1 → 5

| | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Q1 | The Engineering Formulae booklet helps me learn engineering modules better. | | | | | |
| Q2 | The booklet helps me understand (relate) better how mathematical concepts are applied in engineering modules. | | | | | |
| Q3 | It is very convenient for me to refer to the booklet whenever I need the formulae. | | | | | |
| Q4 | With the help of the booklet, I am motivated to learn engineering mathematics. | | | | | |
| Q5 | I think I will use the booklet in my work or in my further studies after graduation. | | | | | |

Any other comments

Thank You