

# AN INTRODUCTORY PROJECT IN FINANCIAL ENGINEERING

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## **Abstract**

A new introductory project has been developed with the aim to motivate first year undergraduate students. To achieve this, the project has strong connection to real life applications, applies mathematics that are taught during the first year and the project is flexible enough to always allow a successful delivery. The application is in finance where the students without prior knowledge determine equity statistics, optimal portfolios, portfolio risk and the properties of a derivative portfolio in the Reuters financial real-time system PowerPlus Pro. To determine this, the students have to apply knowledge from linear algebra, statistics, optimization, and calculus. Our experience is that the students have been highly motivated since it is evident that this is an important problem that they actually can solve with their recently obtained knowledge. It can be concluded that this introductory project in Financial Engineering can be successfully used in the CDIO framework for first year undergraduate students.

*Keywords: CDIO, finance*

## **Introduction**

Many undergraduate educational programs in engineering and science start by studying elementary courses in mathematics. These courses have often small relation to real life applications. Many students want to see how they can use the mathematical tools in the future and how it can be applied in real life. Therefore, some educational programs in engineering and science include a course early in the program in order to motivate the students why they should study elementary courses in mathematics and what these courses give in the long run. These courses often include a larger project, where the two main aims are to introduce basic methods for project work and to give the students a perspective on engineering and their future role as an engineer. Hopefully, by introducing this course in an early stage of the educational program, the students will understand the meaning with all the hard work with elementary courses. The project included in the course is completed on a topic of relevance.

A couple of years ago, we had a project running where the remarks from the students after the project were that they really liked the project, but they did not see how this could be used in real life, i.e., the project did not give them any motivation for the studies of elementary courses in mathematics. It is clear that the introductory project should not only train them in working within projects, it should also be motivating and stimulating for them to work with. Thus we set out to create a new project that better deal with the changing role for introductory courses.

The project should include solving a real life problem, where the students can apply their skills in mathematics and programming, i.e., we want to illustrate the importance of for example linear

algebra, calculus, and programming in real life. To combine the two often conflicting goals of not exceeding the total time with a successful completion of the project requires a project with large flexibility where shortcuts are possible. This is especially important since it is often not possible to control the students' previous knowledge early in the program. This new project concerns finance including mathematics such as algebra, calculus, optimization and mathematical statistics together with programming in Visual Basic for Applications (VBA). The idea is to let students, without prior knowledge in the financial area, create a product that could be used by investors.

In the area of finance demand for people with experience of advanced mathematics and programming has grown in recent years. In finance uncertainty has to be dealt with. What has happened is that the methods that have been developed in the area of engineering are nowadays applied in finance.

The Applied Physics and Electrical Engineering program at Linköping University follows the CDIO syllabus [1] for project work during the whole five years, and this project is included in the first course including CDIO. The most important aims with this course are to work in a project according to a project model, plan a project work from a given specification, make and modify a project plan, find knowledge of relevance to solve the project tasks, relate basic concepts in mathematics, physics and electronics to engineering work, collaborate to make the work in a project group successful, administrate and document an ongoing project work, and present the results from the project work both orally and in a written report. The CDIO syllabus gives the students a well defined documentary to follow in order to fulfill many of these aims with the course.

### **Motivation of the project from a motivation theory perspective**

In motivation theory it is distinguished between extrinsic motivation (source is outside the individual) and intrinsic motivation (source is within the individual) [2]. Since the project is made within a course the extrinsic motivation is already in place in terms of student credits for the course etc. What we can do is to create a project that gives intrinsic motivation. The objective for the course is to motivate the students for engineering studies [3]. To accomplish this we would like to motivate the students to study the basic courses in mathematics. The students usually find the first mathematical courses very theoretical and they cannot see the necessity of this knowledge. This realization usually comes in the third or fourth year when the applied courses are studied that applies the mathematics. Motivation for engineering studies can be increased by providing additional goals to the students [2]. Therefore, to motivate the students for engineering studies we must therefore already in the first project show how the mathematics they learn during the first years can be used to solve real life problems. When the students see that mathematics can be applied, they can understand that the goal is not to learn mathematics but rather to actually be able to solve problems, and thereby add this as an intrinsic goal for learning mathematics. To achieve this, the project is designed to use mathematics that is implemented with a programming language in a real life application.

The main features in this finance project, which is described in detail in the next section, are, asset properties, risk management, optimal investments and options. Given historical data, it is possible to calculate a number of financial values, for example variance and Value at Risk

(VaR). The values are a help and guidance to the user to, for example, chose which stocks to buy and which portfolio that has the best revenue compared to its risk. In order to evaluate these values one need to implement matrix inversion, matrix multiplication and understand some mathematical statistics such as expected value and standard deviation. Some of the areas are completely new and some are included in courses which are given at the same time as the project is running. This really motivates the students to study the elementary courses in mathematics. To solve the project is not trivial, since it requires knowledge which is above their current level. It is however attainable within the prescribed time limit. E.g. [4] stress that it is important to set goals that are attainable and align appropriately with one's skill set and ability. Our experience is that the students, once finishing the project, have much higher confidence since they know that they can solve a "difficult" applied project (to achieve this it is also necessary that the project is flexible).

In the project, the students work with a program from one of the major suppliers of financial data, Reuters. By using an Excel interface created by Reuters, PowerPlus Pro, real time financial data can be accessed. This provides the students with the same data that all other investors in the financial markets have access to. This huge amount of data is used to compute the above mentioned values. When the students work in Excel they have access to VBA and most of the computations are programmed in this language. Reuters is an expensive program to use. However, Linköping University has a number of Reuter licenses and a specific finance room where students taking finance courses have access. The students that participate in this project have also given access to this room. This further increase the motivation of the students since they use a program which many investors actually use to do their daily investment decisions, and they implement a program that contains some of the functionality that an investor would use.

### **Project description**

The objective of the project is to determine the optimal investments in a portfolio consisting of equity (shares), determine the risk of the portfolio and finally to determine the risk and properties of a derivative portfolio. To determine the optimal portfolio the students have to use knowledge from linear algebra. The optimization problem that is solved is to determine the portfolio with minimal risk given a demanded expected return. This is the well known Mean-Variance model. The students also have to apply knowledge in statistics to calculate the assets expected value and covariance. Given these statistics the risk in the portfolio can be determined. The risk is expressed in terms of Value-At-Risk (VaR). This is a widely used risk measure that given a confidence level determines the maximum loss over a time period. The derivative portfolio contains call and put options on equity. An option gives the owner the right to buy (call option) or sell (put option) the underlying asset at a specified price at a given time. Since the value of the option is related to the price of the underlying asset it is highly relevant to determine how the value of the option change when the price of the underlying asset change, i.e. the partial derivative of the option price with respect to the price of the underlying asset. This value is denoted delta. There are also several other changes that are relevant to the price of the option, these partial derivatives are known as the Greeks.

Let  $S_{it}, t = 0, \dots, T$  denote the historical prices for equity,  $i$ , from day 0 to  $T$ . The daily returns for equity can be calculated as  $r_{it} = S_{it} / S_{it-1} - 1, t = 1, \dots, T$ . The subsequent notation and presentation of the material is standard in financial engineering, see e.g. [5].

### **Equity statistics**

Equity prices are often assumed to follow a stochastic process. The first step is therefore to determine the statistical properties of the equities. On average the yearly historical return has been

$$\mu_i = \frac{1}{T\tau} \sum_{t=1}^T r_{it}, \quad (1)$$

where  $\tau = 1/250$  if there are 250 trading days in one year. The risk in equity prices is measured with yearly variance and is determined as

$$\sigma_i^2 = \frac{1}{(T-1)\tau} \sum_{t=1}^T (r_{it} - \mu_i\tau)^2. \quad (2)$$

To measure the relationship between different equity prices the yearly covariance is used

$$\sigma_{ij} = \frac{1}{(T-1)\tau} \sum_{t=1}^T (r_{it} - \mu_i\tau)(r_{jt} - \mu_j\tau), \quad (3)$$

where  $\sigma_{ii} = \sigma_i^2$ . The students can be introduced to matrices and vectors by referring to the storage of equity prices in spreadsheets (Excel). The expected return and covariance can be calculated either with Excel's built in functions or with VBA. The expected return is stored in a vector  $\mu = (\mu_1 \dots \mu_N)^T$  and the covariance in a matrix

$$C = \begin{pmatrix} \sigma_{11} & \dots & \sigma_{1j} & \dots & \sigma_{1N} \\ \vdots & \ddots & \vdots & & \vdots \\ \sigma_{i1} & \dots & \sigma_{ij} & \dots & \sigma_{iN} \\ \vdots & & \vdots & \ddots & \vdots \\ \sigma_{N1} & \dots & \sigma_{Nj} & \dots & \sigma_{NN} \end{pmatrix}. \quad (4)$$

It will now be assumed that the historical estimates of expected return and covariance also will be realized for future equity returns.

### **Portfolio properties**

An investor that invests the share  $w_j$  of the total wealth in equity  $j$ , with the total investment

$w = (w_1 \dots w_N)^T$ , can determine the expected portfolio return as

$$\sum_{j=1}^N \mu_j w_j = \mu^T w. \quad (5)$$

The risk of the portfolio, measured in terms of variance, is

$$\sigma^2 = \sum_{i=1}^N \sum_{j=1}^N w_i \sigma_{ij} w_j = w^T C w. \quad (6)$$

This introduces the students to matrix multiplications, it can be implemented either with Excel's built-in functions or they can write their own in VBA.

### ***Risk management***

Value at Risk (VaR) is a risk measure that has gained tremendous importance in financial markets [6]. VaR transforms the somewhat obscure measure of *volatility* (standard deviation) to a measure in dollars that management can understand. E.g. 3 day VaR at 95% level could be defined in such a way that the maximum loss in 3 days with 95% probability is 1 million dollars. Alternatively stated 1 million dollars or more will be lost with 5% probability. If equity returns are normally distributed with the cumulative probability distribution,

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{x^2}{2}}, \quad (7)$$

then  $VaR = W\sigma N^{-1}(\alpha)\sqrt{t}$  where  $W$  is the portfolio wealth,  $\sigma$  is the portfolio volatility,  $\alpha$  is the confidence level and  $t$  is the time measured in years. To evaluate the inverse of the cumulative probability distribution can be solved in several different ways, but since time has been short the students have always used the built-in function in Excel.

### ***Optimal investments***

The investments  $w$  corresponds to share of capital that is allocated to different equity and should thus always sum to 100%. Therefore, the following constraint must be fulfilled,

$$\sum_{j=1}^N w_j = \mathbf{1}^T w = 1, \quad (8)$$

where  $\mathbf{1}$  is a column vector of ones. Negative investments (short selling) are also allowed. All the optimal portfolios can be represented with the efficient frontier for investors that only consider expected return and variance. It is sufficient to know two portfolios to represent the entire efficient frontier. An investor that minimize risk solve

$$\begin{aligned} \min \quad & w^T C w \\ \text{s.t.} \quad & \mathbf{1}^T w = 1 \end{aligned} \quad (9)$$

with the optimal solution

$$w_R = \frac{C^{-1}\mathbf{1}}{\mathbf{1}^T C^{-1}\mathbf{1}} \quad (10)$$

An investor that would like to maximize long term growth solve

$$\begin{aligned} \max \quad & \mu^T w - \frac{1}{2} w^T C w \\ \text{s.t.} \quad & \mathbf{1}^T w = 1 \end{aligned} \quad (11)$$

with the optimal solution

$$w_G = C^{-1} \left( \mu - \frac{\mathbf{1}^T C^{-1} \mu - 1}{\mathbf{1}^T C^{-1} \mathbf{1}} \mathbf{1} \right). \quad (12)$$

The efficient frontier can now be described by the affine combination of the two portfolios,  $w_E = \lambda w_G + (1 - \lambda) w_R$  where  $\lambda \in \mathfrak{R}$ . Depending on the students knowledge they can actually implement the matrix inversion from pseudo code in VBA or they can use the built in functions in Excel.

### ***Sensitivity analysis of option prices***

A call option gives the owner the right to buy the underlying asset for  $K$  dollars in  $T$  years. Using Black and Scholes theories [5] options can be priced given the continuous interest rate  $r$ , the price of the underlying asset,  $S$ , and the volatility of the underlying asset,  $\sigma$ . A put option gives the owner the right to sell the underlying asset. The price of call option,  $c$ , and put option,  $p$ , is given by

$$\begin{aligned} c &= S N(d_1) - K e^{-rT} N(d_2) \\ p &= K e^{-rT} N(-d_2) - S N(-d_1) \end{aligned} \quad (13)$$

where

$$\begin{aligned} d_1 &= \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\ d_2 &= \frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}. \end{aligned} \quad (14)$$

To better understand the behavior of option prices the Black-Scholes equation is often studied through its partial derivatives. The most common is the partial derivative of the option price with respect to the price of the underlying asset ( $\Delta$ ). This describes how the price of the option change when the price of the underlying asset change. For a call option  $\Delta$  can be calculated as

$$\frac{\partial c}{\partial S} = N(d_1) \quad (15)$$

and for a put option

$$\frac{\partial p}{\partial S} = N(d_1) - 1. \quad (16)$$

Other partial derivatives can also be calculated, all which describe the sensitivity of changes in different parameters. These parameters are called the Greeks. The other Greeks can be added as an additional requirement to the student project.

### **Solving the project using the CDIO framework**

This specific project is given to the Applied Physics and Electrical Engineering program at Linköping University to the first year undergraduate students as a part of the course Engineering project, Y, 6 ECTS credits. The project course is a 6x120 hours course where groups of at least six students do projects according to the CDIO syllabus. The Applied Physics and Electrical Engineering program follows the CDIO syllabus through a number of courses, where this course is the first of at least three courses using the CDIO syllabus. In the first course (and especially the project), where the students are getting familiar with CDIO, focus must be concentrated on both the project itself and teach how to run project by using CDIO. For each project there is a customer, project manager, and a supervisor. The customer in this project is a faculty member of the department. The supervisor, also a faculty member, supports the group with mathematical and technical issues. The project manager is one of the students in the project group. In the beginning, the customer approves the original project plan. The final product is in the end evaluated against the requirements in the specification. Requirements that the group might not be able to meet will be discussed and negotiated with the customer. The students are often struggling with the project plan and they often feel that there are too much work spent working with the project plan. However, in the end of the project, all students agree that the time spent with the project plan early in the project was well spent. The project group reports the ongoing project to the customer at a specific number of toll gates which is given early in the project phase. At each toll gate, the customer decides if the project should carry on into the next phase. Meetings between the group and the supervisor are held on average every second week. Within the groups, each student has one area of responsibility, where the areas of responsibilities in this project are project manager, documents, technical expert in optimal investments, technical expert Reuters, technical expert risk management, and technical expert option. The project ends with a final report, a technical document, and an oral presentation of the project.

### **Project results**

The students get access to a basic portfolio management system that has been written in Reuters PowerPlus Pro (an extension to Excel). The system use historical transactions (figure 1) to build the current portfolio with VBA.

20080408	Deposit										2 000 000
20080409	Sell	ABB.ST	Equity	500	163,5						81 750
20080409	Buy	AZN.ST	Equity	600	242						-145 200
20080409	Sell	ERICb.ST	Equity	33 300	12,37						411 921
20080409	Buy	SHBa.ST	Equity	3 400	183						-622 200
20080409	Buy	SKAb.ST	Equity	2 600	114						-296 400
20080409	Buy	SKFb.ST	Equity	6 000	122,75						-736 500
20080409	Buy	VOLVb.ST	Equity	6 800	98						-666 400

Figure 1. Transaction history

By pushing the “Update portfolio” button (figure 2) the current portfolio is updated. In PowerPlus Pro the current holdings are shown together with real time information from the financial markets such as equity prices, volume, interest rates and current value of the investments (figure 2).

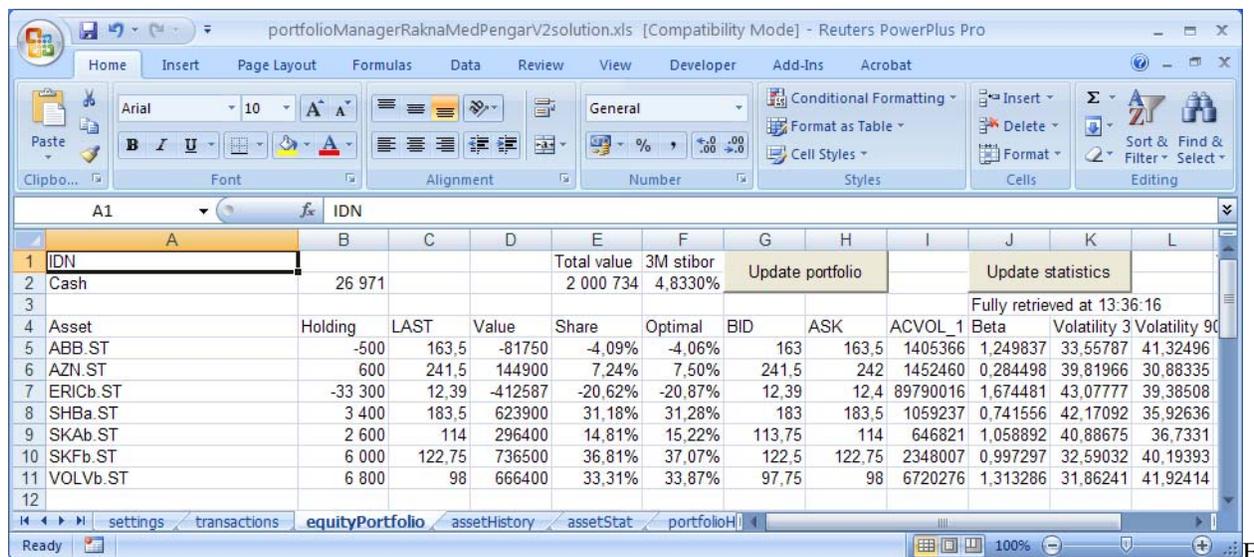


Figure 2. Portfolio view

At the same time the historical prices for the equities in the portfolio are loaded from Reuters (figure 3). From these data the historical expected return and covariance are calculated (equation 1 and 3). Help functions have been written in VBA that read the historical data from Excel and write the expected return and covariance to Excel. Thus it is only necessary for the students to write a relatively simple VBA function that contains loops to calculate values from matrices. So far there has always been at least one student with previous programming experience. Even if there were no previous programming experience in the group, this can be learnt in short time. There is always also the option to use Excel’s built in statistical functions.



Figure 3. 10 years of daily historical close prices

The next step is to calculate Value at Risk (table 1). This introduces relatively simple algebra calculation in VBA (equation 6). Since Excel has a function that can calculate matrix multiplications and if the students get stuck in writing the functions, then there is always the possibility to fall back to these by either calling them from VBA or straight from Excel.

Table 1. Calculation of Value at Risk

Confidence	95%
Days	3
VaR	89 578

The efficient frontier requires more advanced algebra calculations since matrix-matrix multiplication, matrix inversion and affine combinations are required (equation 10 and 12). Since the students have little experience in using matrices this part is viewed as the largest hurdle. But given time and some tips and ideas from the supervisor the students can sort out all the details. A few project groups have even written their own functions for inverting matrices in VBA. The usual case is otherwise to use Excel's built in function for matrix inversion. The result is an efficient frontier, from which a suitable portfolio can be selected (figure 4).

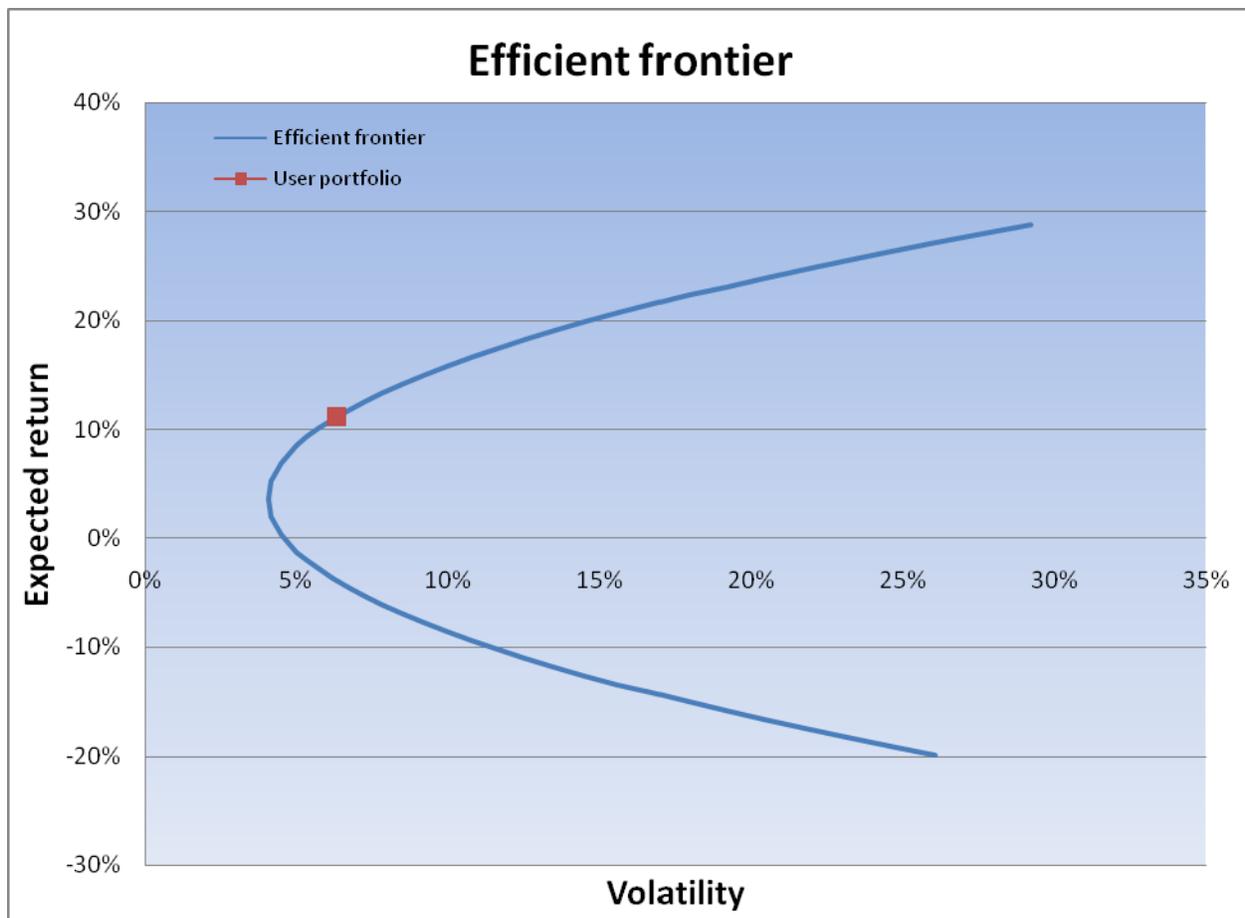


Figure 4. The efficient frontier and the selected portfolio

The final step is to calculate the properties of an option portfolio (equation 14, 15 and 16). The students have not used their calculus knowledge to derive the Greeks (even though this would be feasible). These relatively complicated expressions can be found in relevant literature. Even though they are complicated, they are not very difficult to implement for any student at the program. However, they do provide important information about options and also show that the results from calculus can be used to provide important information. This together with the experience that algebra can be applied only a few months after the students have begun to study at the program hopefully shows the importance of studies in mathematics. It also shows that programming is an important tool to realize mathematics. Or as one group expressed it “It is fun to apply the mathematics we learn on something that can be used in a more concrete way”.

## Conclusions

This introductory project in Financial Engineering has been used for two years and the results have been very successful. Therefore, it can be concluded that this project can be successfully used in the first year for undergraduate students. Even though the students will study areas that are not necessarily related to finance (there is one profile in financial mathematics) they have successfully delivered products that can be used to solve these problems in real time. The project not only widens the students’ perspective to the area of finance it also gives them opportunity to

use recently obtained knowledge in linear algebra, calculus, statistics, and programming. The students have been highly motivated since it is evident that this is an important problem. When students have intrinsic motivation, it is possible for them to solve projects that require that they learn quite a lot more. The project also works well in the CDIO framework since the project contains several small problems thus making it easy to divide different responsibilities in the project group. Thus the students also learn to work in a group, and efficiently divide the work between them.

The students have found it very inspiring that they are working in an environment that is used in the financial market to process financial data in real time, using only the knowledge gained during the first months of their studies. To create realistic and challenging projects is central for all projects to get fully motivated students. Even though finance is not the students area of study it can be recommended that a project of this type is used, since it trains the students in many important areas and it is also one area where the students could work in the future since the financial industry requires people with engineering background.

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