

# Teaching Mathematics for Civil Engineering Students applying Experiments

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## ABSTRACT

For the last five years at the Aarhus School of Engineering a series of experiments which are closely related to the work as a civil engineer have been developed for supplementing teaching in differential equations and basic statistics and probability theory.

The series of experiments which the students have to perform and report in groups are:

CO<sub>2</sub> concentration: The physical problem is modelled through a first order differential equation and as the scatter is rather high it is a very good basis for teaching statistics. The indoor climate is always a problem in lecture rooms and the students are therefore a priori very motivated. This is the only experiment in the series the student does not have to design and implement for themselves.

The pendulum: Second order differential equations and linearization of harmonic equations is the mathematical topic. Even though the experiment is nothing more than having a mass and a string it is a challenge to perform good measurements. The students need to be inventive and perform well as a group to obtain success.

Dynamics of a chimney: The scope is as for the pendulum but there is also the aspect of structural engineering included.

Stability of a column: Again the mathematical topic is second order differential equations and again the aspect of structural engineering. The design and implementation of the experiment is not straight forward, especially the boundary conditions.

A student satisfaction measurement has been performed and in general the students are satisfied with the learning method. Especially working in groups is appreciated by the students

## KEYWORDS

Mathematics, achievement results, integrated learning, teamwork, gender differences

## INTRODUCTION

Teaching mathematics for civil engineering students at the beginning of their study is a special challenge. Most students have only limited knowledge of what the work as a civil engineer implies and even though they have some idea that mathematics is important for their study a large percentage ends up having problems with basic mathematics. As a result

students are very focused on why they need to use so much time and effort on studying this topic.

For these reasons at Aarhus University School of Engineering on the second semester a special course for assisting in the learning the theories based on experiments has been developed. The course also includes traditional experiments in structural mechanics and hydraulics. We call the course “Theme Assignments” and in this paper only the theme Mathematics is considered.

At Aarhus University School of Engineering there are two ways of entering the studies. The traditional way of following 12-13 years of studies and the second way is to attend 9-10 years of undergraduate school have and education as a craftsman and then after some years working as a craftsman attending a special one year long preparatory study where Mathematics, English, Danish and Physics are the main topics. About 20%-25% of the students are craftsmen. About 30% of the students are female.

The courses in Mathematics are mainly taught by civil engineers at Aarhus University School of Engineering thus emphasizing on a practical approach. In the cases of the Theme Assignments it is the same teacher as for the mathematics course. The contents and learning objectives can be seen in Table 1.

Table 1  
Contents and learning objectives for “Mathematics” and the “Theme Assignments”

<b>Learning objectives</b>
When the course is completed, the student is expected to be able to: <ul style="list-style-type: none"> <li>• Create mathematical models describing physical phenomenon’s and afterwards evaluate the validity of the calculation model.</li> <li>• Evaluate the most significant aspects by designing models</li> </ul>
<b>Contents</b>
<ul style="list-style-type: none"> <li>• Complex numbers</li> <li>• Taylor series</li> <li>• Second order differential equations</li> <li>• Programming</li> <li>• Numerical methods including solving equations and differential equations</li> <li>• Statistics</li> <li>• Graphical presentation of data</li> <li>• Stochastic variables</li> <li>• Cumulative Distribution function and probability distributions</li> <li>• Characteristic values and confidence intervals</li> </ul>

Within the first week of the semester the student will be divided into groups of 3-5 persons. Each group will have to work together during the whole semester and prepare, perform and report the experiments together.

In the beginning of the semester the students are introduced to basic measuring systems and devices. As soon as the necessary theory has been thought they are introduced to one or more experiments. Also, a six page document outlining how to write an engineering report, doing quality control and a loose formulated outline of each experiment is given to the students [1].

Before the students can perform the test they have to write an initial report were they outline what they intend to do including the basic theory and what parameters they intend to vary. This description is checked by the teacher.

The reports form the basis for an oral exam with duration of 20 minutes.

## THE EXPERIMENTS

In the following a short description is given of the experiments the students has to perform in connection to the Theme Assignments.

### **CO<sub>2</sub> concentration**

For a civil engineer modelling of the indoor climate is of outermost importance in order to be able to ensure a satisfactory living and working environment. The CO<sub>2</sub> concentration itself is seldom a problem but it is a very good indicator on the amount of other aerosols which has an influence on the indoor climate, however it is very easy to measure and there are recommended limits that will insure a good indoor environment. By considering the incoming, outgoing and produced CO<sub>2</sub> in a room the so called dilution ventilation equation arises:

$$\frac{dc(t)}{dt} = \frac{G}{V} + c_e n - c(t)n \quad (1)$$

Were  $c(t)$  is the concentration of CO<sub>2</sub>,  $n$  is the number of times per hour the air in the room with volume,  $V$ , changes,  $G$ , is the amount of CO<sub>2</sub> generated in the room (from the students) and  $c_0$  is the outdoor concentration. This is a first order differential equation. Assuming that the concentration for  $t=0$  is  $c_0$  the analytical solution is given as [2]:

$$c(t) = c_e + (c_0 - c_e)e^{-nt} + \frac{G}{nV}(1 - e^{-nt}) \quad (2)$$

If  $G$  is assumed to be a function of time the equation becomes nonlinear and a numerical technique has to be used. There is a lot of scatter in the measured results and therefore this exercise also forms a god basis for performing statistical analysis.

For the experiment TinyTag Units [xx] are used for data acquisition, thermometers and a CO<sub>2</sub> measuring device. Se Figure 1, for a test setup.



Figure 1. The sensors for CO<sub>2</sub> and temperature measurements placed on the teachers desk.

### **Pendulum**

For a civil engineer a pendulum is not very relevant but for understanding the mathematical topics of second order differential equations and linearization of harmonic equations it is ideal. Even though the experiment is nothing more than having a mass and a string it is a challenge to perform good measurements and applying a constant external load.

The position of a pendulum can be described by the angle to vertical,  $\theta$ . By applying Newton's second law on the mass,  $m$ , of a pendulum attached to a string of length  $L$ , subjected to the gravity,  $g$  and an external constant load  $P_0$  the following equation can be derived [xx]:

$$\ddot{\theta} + \frac{g}{L} \sin(\theta) = \frac{P_0}{m} \quad (3)$$

This is a second order nonlinear inhomogeneous differential equation and the solution can only be found using numerical techniques.

Taylor series is also part of the learning objectives and it can for small angles be simplified to:

$$\ddot{\theta} + \frac{g}{L} \theta = \frac{P_0}{m} \quad (4)$$

Thereby turning it into a second order linear inhomogeneous differential equation. The solution is:

$$\theta = \theta_0 \cos\left(\sqrt{\frac{g}{L}} t\right) + K \quad (5)$$

For the test the students are given a video camera and an introduction to the open source programme Tracker which can convert a point on the filmed pendulum into coordinates making a direct comparison of the model possible [xx].

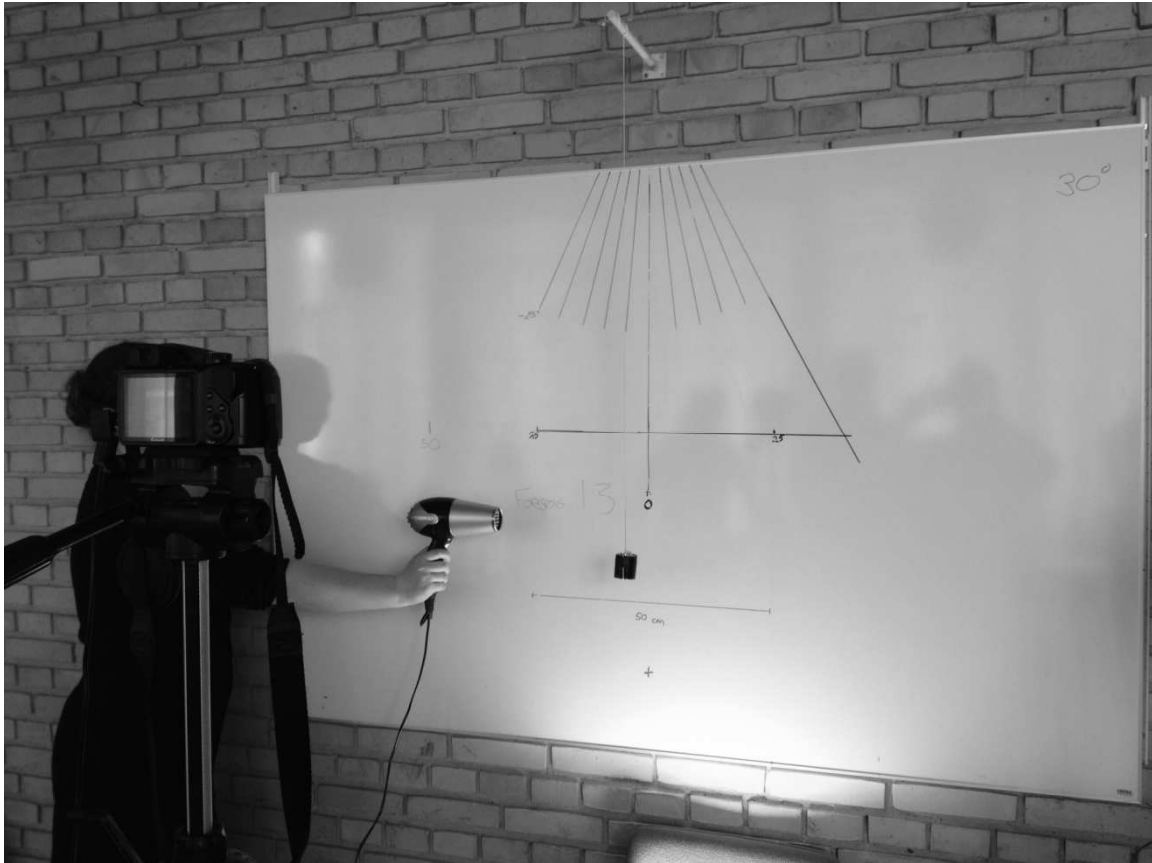


Figure 2. Typical setup for the pendulum experiment where a hairdryer is used to apply the constant load.

### **Chimney**

A typical structure with vibrational problems is a chimney subjected to wind loads. The problem arises when vortices are formed and harmonically loading the chimney perpendicular to the wind direction. The differential equation for the vibrating mass,  $m$ , at the top of the chimney is determined by using d'Alembert's principle:

$$m\ddot{x} + c\dot{x} + kx = C\sin(\omega t) \quad (7)$$

Where  $c$  is the damping of the system,  $k$ , is the stiffness of the chimney and  $C\sin(\omega t)$  is the external load. This is a second order linear inhomogeneous differential equation. The solution is similar to above except when the loading frequency is close to the eigenfrequency of the system.

For this test a tube has been fixed to the floor in the laboratory with a mass and an engine attached to the upper end of the tube. Also a laser measuring the displacements is used. The test setup can be seen in figure 3.



Figure 3. Test setup for the chimney. Displacements are measured with the laser fixed to the wall on the top.

## STUDENT SATISFACTORY MEASUREMENTS

A student satisfactory investigation has been performed. An electronic message was sent to all students studying as civil engineer on the bachelors level. As the students on the fifth semester are in-turns at a company, there were no respondents from that part. The questions asked can be seen in Table 2. About 300 students from the semester 2,3,4,6 and 7 were given 11 questions. 106 choose to do so and of these 31 were females and 20 had a supplemental education as craftsman. All the craftsmen were male.

The results of the answers are given in figures 4.-11. In the histograms the top figure illustrates the total responses together with the responses of the craftsmen. In the figure below the total responses together with the responses of the female are illustrated. In the figure text mean values are given.

The answers are showing a very similar pattern were most students have given the answer 5 and the mean value is about 4.5. The question about teamwork has the highest score and the three questions about the loose formulation, the time used and communication are the lowest.

It should be noted that for the group of craftsmen communication is higher than the overall mean and a little smaller for most of the other questions. For the group of female students the form and the time used is higher appreciated.

Table 2  
The 11 questions asked and the abbreviation used in the figures

<b>Question</b>	<b>Abbreviation</b>	<b>Possible answer</b>
What is your age?	Age	
What is your current semester?	Semester	B2, B3, B4, B6 and B7
Are you a craftsman?	Craftsman	Yes/No
What is your sex	Sex	Male/Female
To what degree did you in general benefit from the Theme Exercises	General	1, 2, 3, 4, 5, 6 and 7
How satisfied were you with this teaching method?	Form	1, 2, 3, 4, 5, 6 and 7
To what extent has the Theme Assignment helped you in understanding the mathematics?	Understanding	1, 2, 3, 4, 5, 6 and 7
How big a support was it for you that the work was performed in groups?	Groups	1, 2, 3, 4, 5, 6 and 7
Has the Theme Assignments improved you in written communication?	Communication	1, 2, 3, 4, 5, 6 and 7
Is there an appropriate proportion between the time used and Var der et passende forhold imellem den tid du brugte og dit udbytte?	Time	1, 2, 3, 4, 5, 6 and 7
The Oplæget til eksperimenterne var meget løst formuleret. I hvor høj grad har det hjulpet dig med at blive problemløser?	Loose formulated	1, 2, 3, 4, 5, 6 and 7

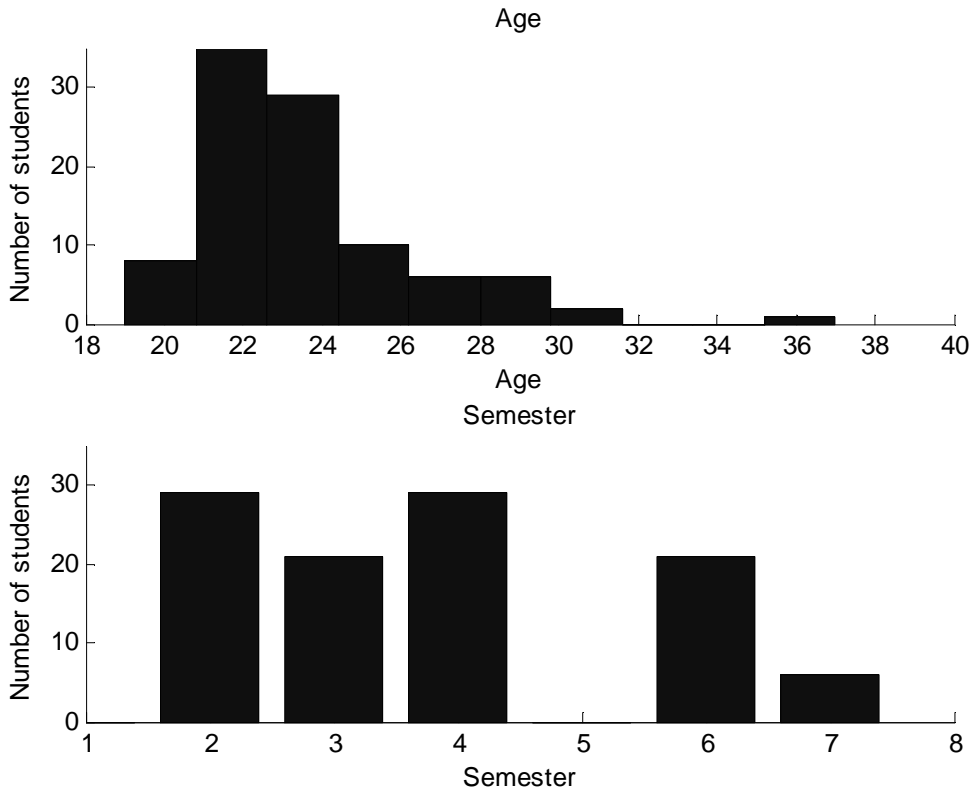


Figure 4. Distribution of age and semester of the respondents.

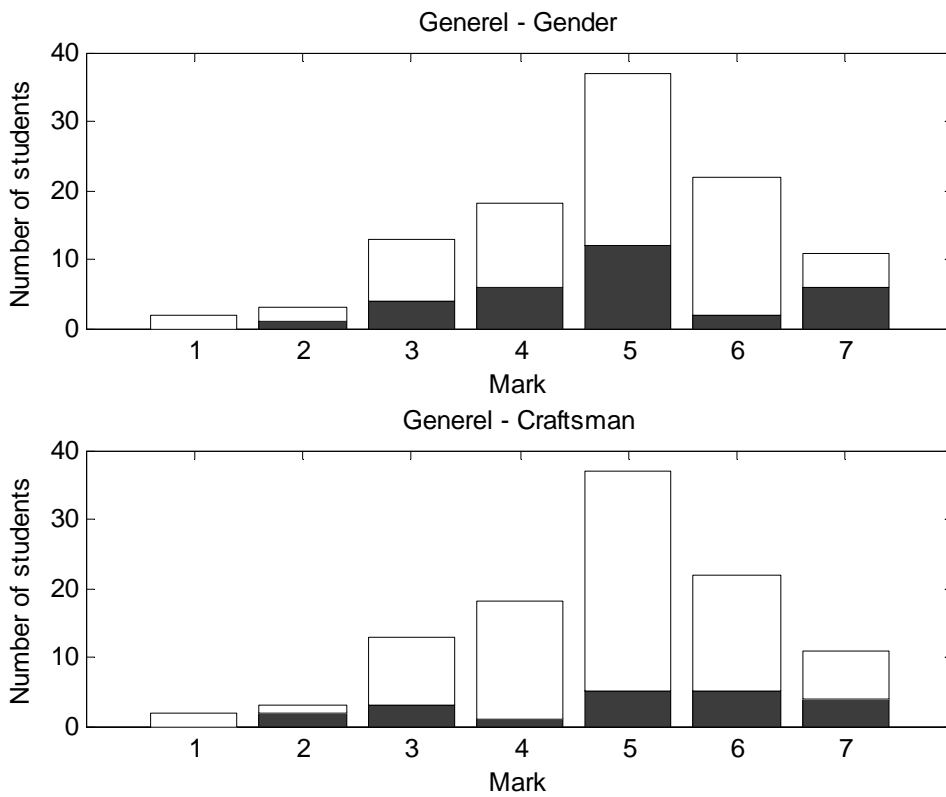


Figure 5. Mean General: 4.84, Mean General Female: 4.90 and Mean General Craftsman: 5.00



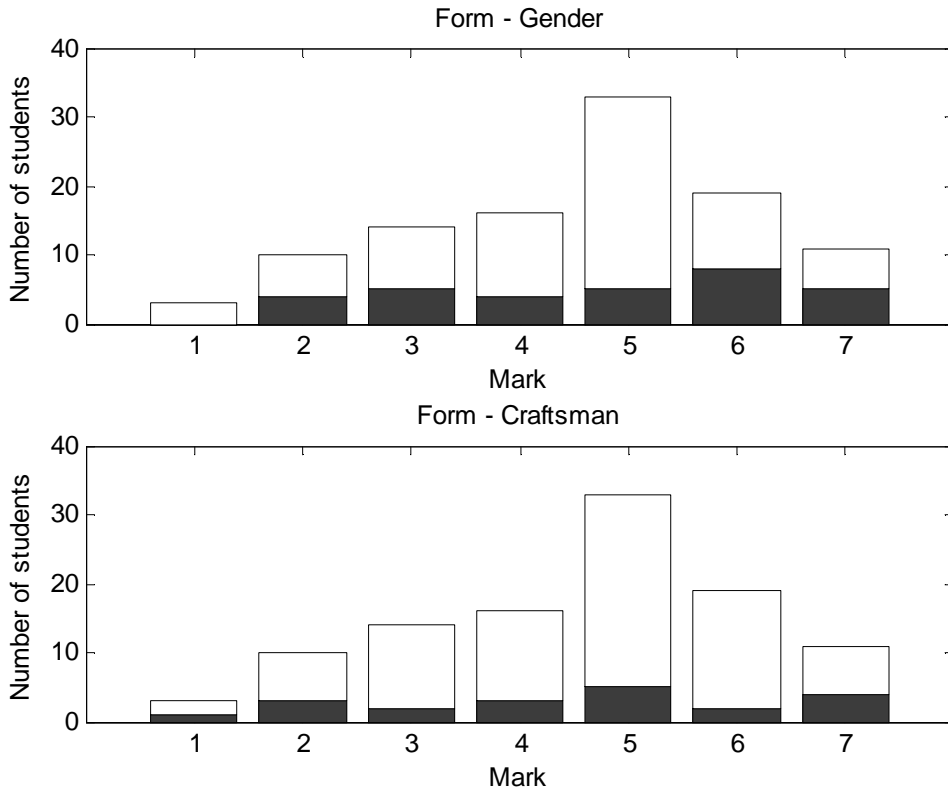


Figure 6. Mean Form: 4.58, Mean Form Female: 4.74 and Mean Form Craftsman: 4.50

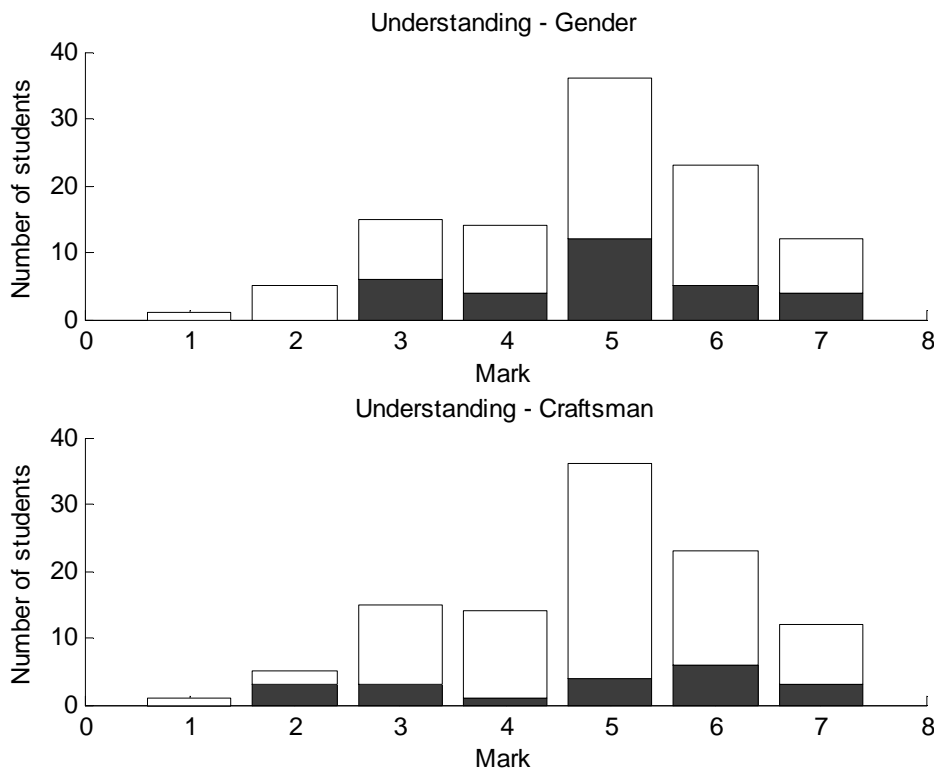


Figure 7. Mean Understanding: 4.85, Mean Understanding Female: 4.90, Mean Understanding Craftsman: 4.85

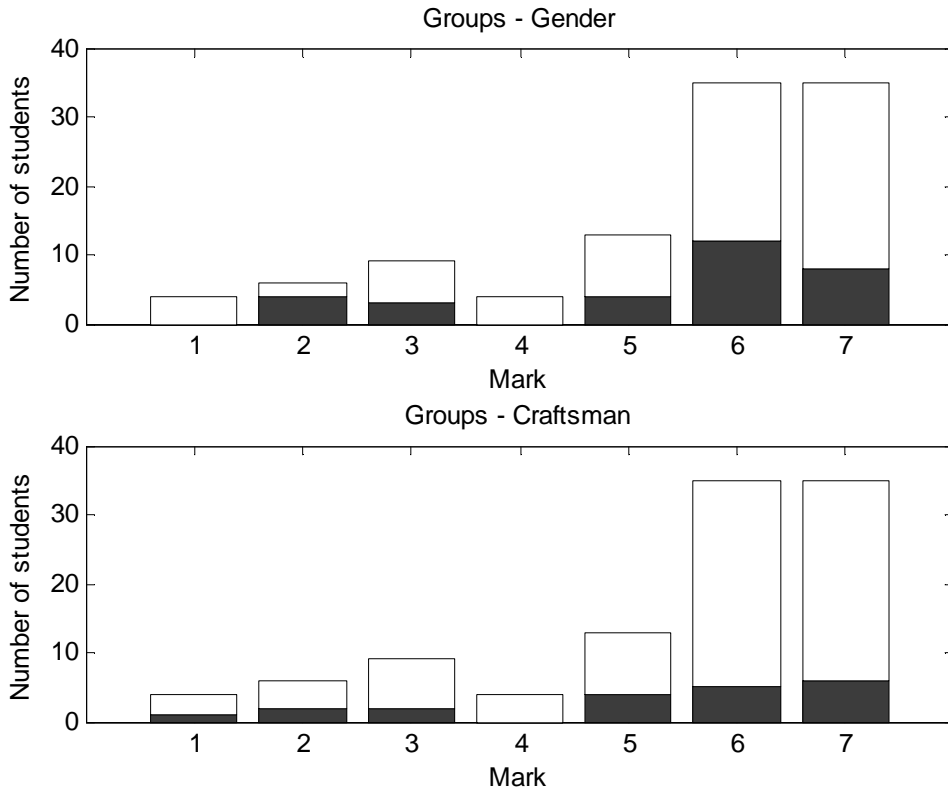


Figure 8. Mean Groups: 5.46, Mean Groups Female: 5.32 and Mean Groups Craftsman: 5.15

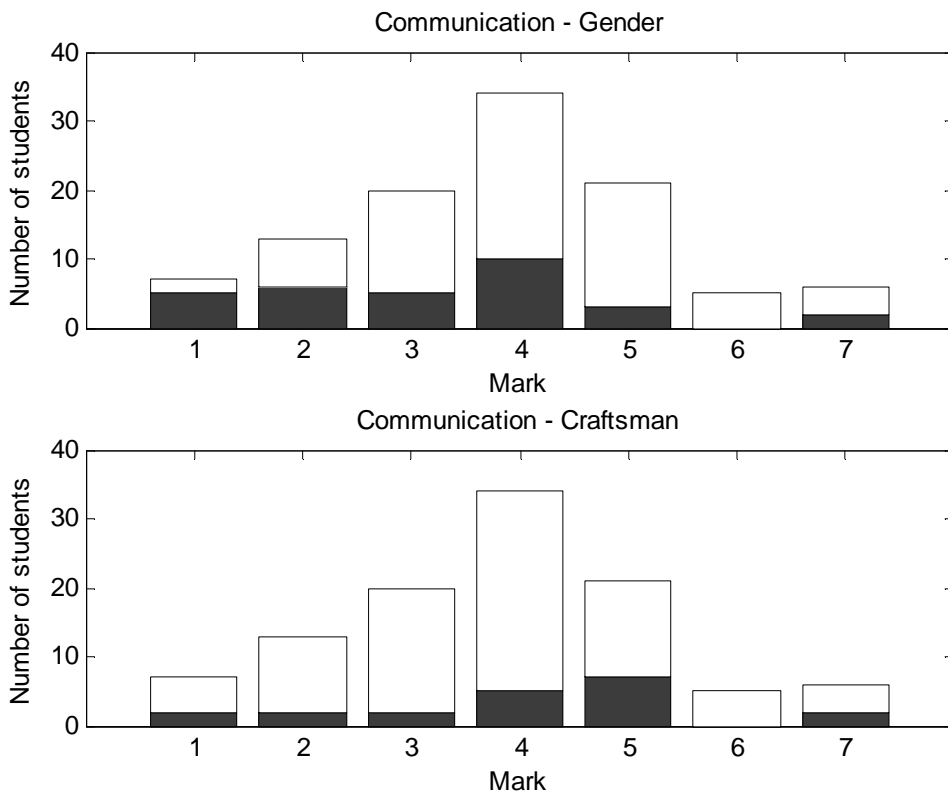


Figure 9. Mean Communication: 3.83, Mean Communication Female: 3.26 and Mean Communication Craftsman: 4.05

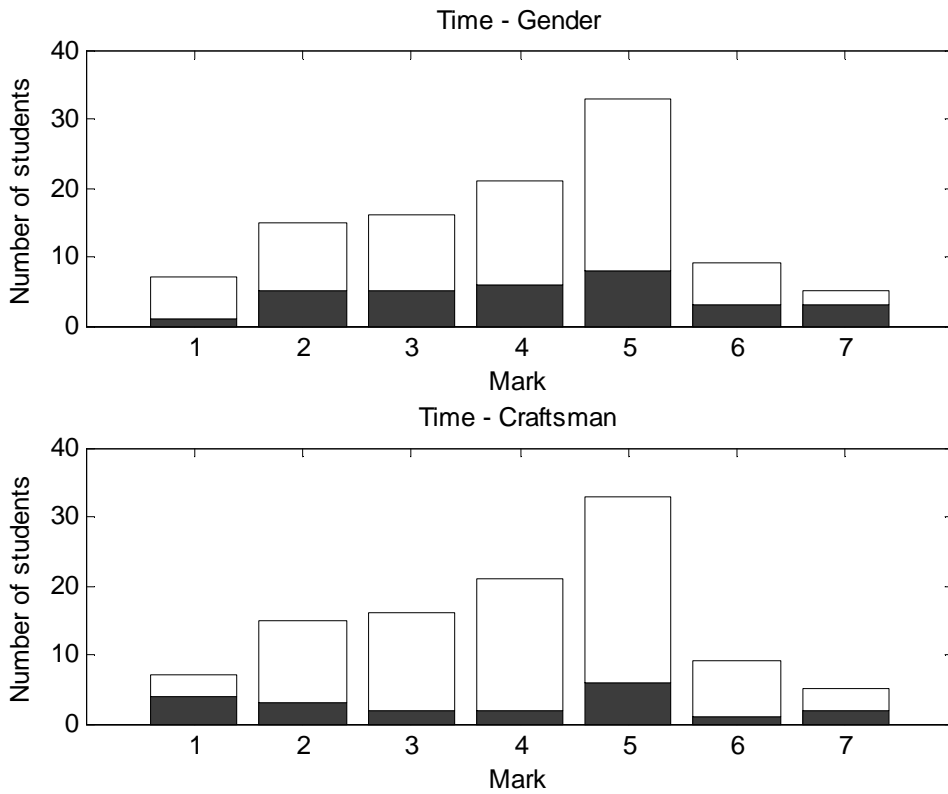


Figure 10. Mean Time: 3.99, Mean Time Female: 4.16 and Mean Time Craftsman: 3.70

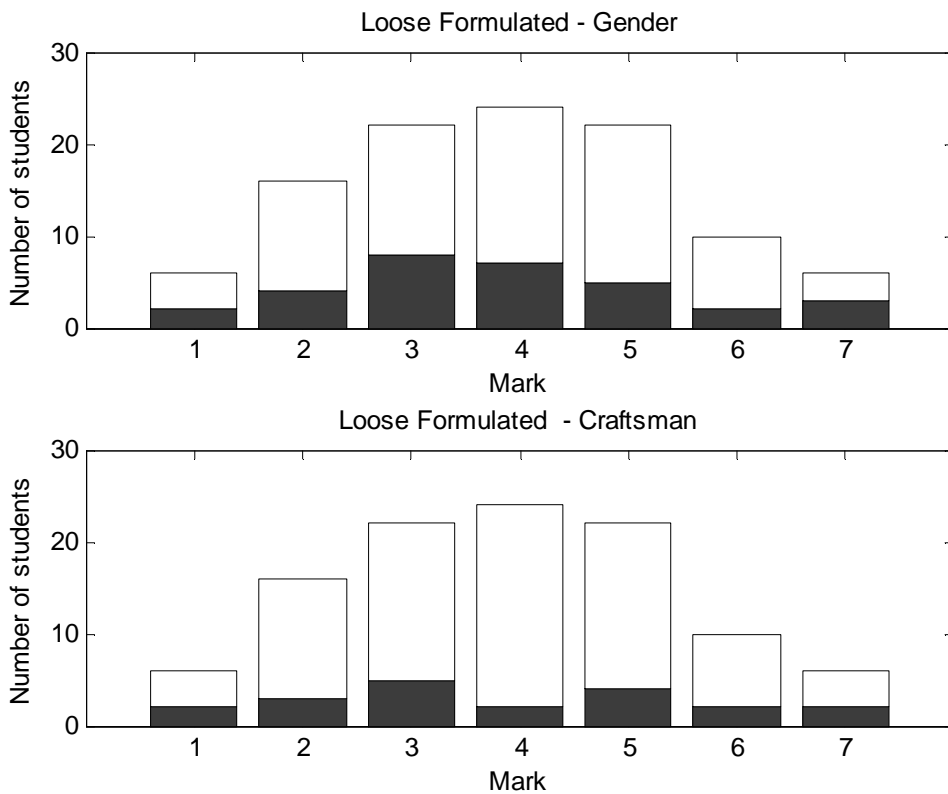


Figure 11. Mean Loose Formulated: 3.89, Mean Loose Formulated Female: 3.87 and Mean Loose Formulated Craftsman: 3.85

## CONCLUSIONS

For the last five years at the Aarhus School of Engineering a series of experiments which are closely related to the work as a civil engineer have been developed for supplementing teaching in differential equations and basic statistics and probability theory.

The series of experiments which the students have to perform and report in groups are: Measuring the CO<sub>2</sub> concentration in a classroom related to the indoor climate in buildings, the pendulum and a steel tube fixed the floor related to vibration problems of structures like chimneys.

A Student satisfactory investigation has been conducted and in general the students are satisfied with this type of teaching model. The method has in general helped the students in understanding the theory and working in groups is appreciated.

There is not a real significant difference in in gender appreciation or if the student has a background as a craftsman.

## REFERENCES

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### ***Biographical Information***

Jens Peder Ulfkjaer is a Civil Engineer in Structural Engineering and Associate Professor at Aarhus University School of Engineering. His research interest is in fracture of concrete at different loading rates including very high strain rates. His scholarly interests are in implementing experimental activities as an active part of the learning process

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