RESEARCH AND PRACTICE OF THE PROJECT-DRIVEN LINEAR ALGEBRA PRACTICE TEACHING MODEL

Jianwei Du, Shuxia Wang, Qinxian Shi, Xinhong Liu

Zhiyuan College of Beijing Institute of Petrochemical Technology

ABSTRACT

Linear algebra courses are the foundation of engineering disciplines related to artificial intelligence and robotics, and its related knowledge has a wide range of applications in image processing, machine learning, mechanical arm movement, etc. To distinguish from the traditional linear algebra teaching mode, which emphasizes theory and neglects practice and application, and learns from the engineering CDIO education concept, we carry out project-driven linear algebra practical teaching in the linear algebra course. By designing practical projects and assigning practical tasks, students form project teams and complete the project tasks in a specified time through division of labor and cooperation. Teachers design corresponding assessment evaluation standards, and students in the group present and publicize the methods and results of completing the task. Through self-assessment, group evaluation, and teacher evaluation methods, practical achievements are obtained. In the practice of the project-driven linear algebra practical teaching model, students not only deepen their understanding and application of linear algebra concept knowledge, but also enhance their practical hands-on ability, improve their team spirit and communication skills, understand the application prospects of linear algebra courses, and lay the foundation for future study in artificial intelligence and related fields.

KEY WORDS

Linear algebra courses, Engineering education, Project-driven teaching, CDIO Standards: 1,2,3,7,8,10

INTRODUCTION

Linear algebra, along with mathematical statistics, furnishes much of the theoretical underpinning for the development of computer science and engineering, such as image processing, signal processing, machine learning, artificial intelligence, etc., and provide tools for solving many practical problems, such as optimization problems, image processing, network optimization, etc. In summary, linear algebra is a very important branch of mathematics. It has a wide range of applications in mathematics, computer science, physics and other fields. It is the foundation of many important theories and practical problems. As an important basic mathematics course, the traditional method of teaching linear algebra courses is still to talk about definitions, theorems, derivation proofs and calculations. Students often feel boring, abstract, and difficult to understand some conceptual theories. Especially they cannot see how linear algebra is applied. Therefore, from the perspective of teaching effectiveness, students are not motivated to learn. The application of linear algebra only stays in the first lesson—the introduction class. With the advent of the era of artificial intelligence and big data, technology updates are getting faster and faster. The basic role of linear algebra applications is becoming more apparent. The teaching mode of linear algebra courses urgently needs to be updated.

According to the CDIO initiative, its goal is to provide students with an engineering education that emphasizes engineering fundamentals and is based on the conception design implementation operation (CDIO) background environment of real-world products, systems, and processes. (Malmqvist, J. (2020))This education aims to cultivate students' solid engineering fundamental theories and professional knowledge, and through close integration with society and enterprises, enable students to learn and practice in real engineering fields, thereby improving their engineering literacy and comprehensive abilities. The linear algebra course can draw on the CDIO educational philosophy of engineering education as a basic course for engineering education, integrate practical aspects into teaching, target real problems, allow students to actively conceive and design solutions to problems, understand linear algebra knowledge in the process of solving problems, bridge the gap between theory and reality, and improve students' engineering literacy and interdisciplinary comprehensive abilities.

There are multiple teaching methods that can be used to achieve the CDIO standards, among which project-based teaching is a teaching method that focuses on projects, takes students as the main subjects, and is guided by teachers(Zarestky, J. (2022)). Based on teaching objectives and students' actual situations, teachers design a series of projects related to practical engineering problems to enable students to master knowledge and skills in the process of solving problems. (Lehmann et al., 2008) Project-driven teaching can stimulate students' interest and motivation in learning, cultivate their teamwork and problem-solving abilities, and also promote interaction and communication between teachers and students. (Bernhard, J. (2020)) It cultivates students' sustainable development abilities in project-based teaching. Due to the wide applicability of linear algebra, it provides materials for practical teaching of linear algebra. It is feasible to explore practical problems and cases that can be applied as practical content for linear algebra courses, and carry out project-based practical teaching of linear algebra.

The following sections introduce the background for offering a linear algebra course, the methodology, results, discussion, and analysis of project-driven practical teaching of linear algebra. Finally, the conclusion is presented.

Background

The linear algebra course involved in this article is offered to second-year undergraduate students in the Zhiyuan Experimental Class and the School of Information Engineering at our university. The total credits of students in four years should not exceed 173 credits, and the professional education credits should not exceed 120 credits. Linear algebra is a fundamental professional course with 48 class hours and 3 credits. The majors that take this course include intelligent computing, intelligent manufacturing, data science and big data technology, computer science and technology, electrical engineering and automation, and internet of things engineering.There are three teaching classes, each taught by a different instructor.The course objectives and evaluation methods are shown in Table 1 and Table 2 respectively.

Table 1. Course Objectives of Linear Algebra and Supporting Graduation Requirements

Linear Algebra（MATH208）		
Course Objectives	Course objective description	Support strength for graduation requirements(H M L)
Course Objectives 1	Through the study of this course, students will be able to describe concepts such as the number of rows and columns, matrices, systems of linear equations, vectors, vector groups, similar matrices, quadratic forms, etc. They will also be able to identify and explain relevant theories related to matrices, systems of linear equations, vector theory, similar matrices, and quadratic forms, and express and infer relevant conclusions.	H
• Course Objectives 2	Through the identification, comparison, and deduction of the learned linear algebra knowledge, summarize, reorganize, and deal with related problems; design problem-solving ideas for some practical or mathematical problems, compile MATLAB commands, and combine relevant theoretical knowledge to draw conclusions.	M

Table 1 outlines the detailed descriptions, evaluation criteria for the two Course objectives of the linear algebra course. Table 2 outlines evaluation criteria and grade proportion. Among the evaluation criteria, the experimental component accounts for 10% of the grade(The dotted line section). Prior to adopting the project-driven practical model, this experimental content primarily involved using Matlab commands for determinant calculations, matrix operations, solving linear equations, finding eigenvalues and eigenvectors, and addressing some simple application problems. However, these were not authentic real-world problems. In the era of artificial intelligence and big data, these experiments have become outdated, particularly for majors in the School of Information Engineering that are highly correlated with professional graduation requirements.

The research content of this article focuses on improving the previous teaching model related to the experimental component of the evaluation criteria by adopting a project-driven practical teaching model. The following section introduces the main methodologies of the project-based practical teaching model.

Table 2.Evaluation criteria and grade proportion

Order number	Course objectives	Evaluation criteria and grade proportion(%)					Grade proportion/Percentage of grades(%)
		Classroom test(10%)	Home work (10%)	Experiment (10%)	Periodic assessment/test (10%)	Final exam (60%)	
1	Objectives 1	10%	10%		8%	42%	70%
2	Objectives 2			10%	2%	18%	30%

Total	10%	10%	10%	10%	60%	100%
Final assessment format		☑Closed-book written examination ☐Open-book written examination ☐Course paper (design) ☐Course report ☐Practical computer operation ☐Experimental operation ☐Additional items（Please indicate).				

Methodology

Project-based practical teaching involves teachers assigning open-ended project tasks, with students working collaboratively in groups of 4-6 individuals, freely formed. The teacher designs five practical projects (see Table 3), with each project containing 4-5 tasks to be completed by the student groups within a set time frame of 15 to 30 days. Students are required to conduct literature reviews, design problem-solving approaches, and utilize computer software to achieve the desired outcomes prescribed by the tasks. Through this process, students acquire new knowledge and methodologies, gaining a better understanding of linear algebra concepts. Each project provides room for student creativity, such as exploring alternative image compression techniques through literature research and implementing them using software, learning additional image edge detection methods, and explaining observed phenomena using their knowledge of linear algebra. Finally, students summarize and present their findings. The completion of these tasks fosters computational thinking skills, executive abilities, and team collaboration among students during the practical process.

To evaluate the quality of students' task completion, a grading method is adopted that combines intra-group self-evaluation (20%), inter-group peer evaluation (30%), and teacher evaluation (50%) to derive a comprehensive score. The grading criteria are outlined in Table 4.The project design and evaluation criteria correspond to the CDIO standards. Projects are selected from real-life problems, providing students with an emphasis on the Conceive-Design-Implement-Operate (CDIO) process using fundamental knowledge of linear algebra, which aligns with CDIO Standard 1.

Table 3 Linear Algebra Practical Project

Project Name	Relevant Linear Algebra Knowledge Points	Project Code	Project Implementation
Color Image Processing	Addition, Subtraction, and Scalar Multiplication of Matrices	A	First Practice (Choose one from two options)
Geometric Transformation of Plane Figures	Linear Transformation and Matrix Multiplication	B	
Color Image Compression	Eigenvalues and Eigenvectors of Matrices, Similarity Diagonalization of Matrices, Singular Value Decomposition	C	Second Practice(Choose one from three options)
Image Filtering	Inner Product of Vectors, 2D Convolution Operation	D	
Translation and Rotation of Robotic Arm	Continuous Translation and Rotation of Vectors in Three-dimensional Space	E	

The project design revolves around the CDIO process while also emphasizing integration with computer software, in line with CDIO Standard 2. During task completion, students' self-initiative, innovation, and team collaboration abilities are mobilized, consistent with CDIO Standard 3.

As evident from the evaluation criteria, students can integrate their theoretical knowledge with practical applications through project-based learning. They utilize their knowledge of linear algebra to explain real-world problems, establish mathematical models, and align with CDIO Standard 7. Completing practical project tasks requires students to work in teams, actively search for literature, design problem-solving approaches, and engage in hands-on experiences, which corresponds to CDIO Standard 8. Students present their group's project accomplishments through presentations, and their performance is comprehensively assessed based on group self-evaluation, peer evaluation, and teacher evaluation, encompassing knowledge, skills, and attitudes. This holistic approach to assessing students' learning outcomes aligns with CDIO Standard 10.

RESULTS, DISCUSSION, AND ANALYSIS

After the activity, we designed a questionnaire with 12 questions aimed at examining whether students met the requirements of Course Objective 2 during the project-based practical activities and how well they aligned with the expected CDIO standards. The 12 questions and survey results are presented in Table 5 and Figure 1 below.

Table 4 Project Practice Assessment Scoring Criteria

Evaluation Elements	Task 1-4（80 points）				Task 5 （20 points）	
	Completion Status （50）	Image processing can be explained using linear algebra knowledge （10）	Image processing is transformed into a mathematical model（10）	Animation effect （10）	Completion Presentation （10）	Team Cooperation （10）
Inter-group Peer Evaluation （30%）						
Teacher evaluation （50%）						
Intra-group Self-evaluation （20%）						
Comprehensive Evaluation						

Due to the varying number of students in the three classes, Figure 1 shows the distribution of students' responses to each question as a percentage of the total. As can be seen from Figure 1, more than 80% of students in all three classes gave agree or strongly agree responses to all 12 questions, indicating that the project-based practical activities in linear algebra achieved the intended objectives and were positively received by most students.

Table 5 Questions Regarding Project-Based Practical Effectiveness Survey and Corresponding CDIO Standards

Q1.Abstract mathematical problems from simple practical issues.	CDIO Standard 1
Q2. Apply the knowledge of linear algebra learned to solve practical problems.	
Q3.Independently search for information, combine with learned knowledge, and use software to solve practical problems.	CDIO Standard 2
Q4.Able to establish mathematical models for simple practical problems and solve them.	CDIO Standard 3，7
Q5.Able to break down encountered problems into smaller, more manageable issues.	
Q6.Able to face practical tasks with optimism and enthusiasm.	
Q7.Able to apply problem-solving methods learned from completing practical tasks to other problem scenarios.	CDIO Standard 7
Q8.Learned some problem-solving methods from other team members.	CDIO Standard 3，8，10
Q9.Participated in problem-solving and received understanding and encouragement from other team members.	
Q10.Able to correctly evaluate one's own work in completing tasks and hopes to do better in the next practice.	
Q11.Learned some better methods from the work of other groups.	
Q12.Dare to raise reasonable doubts about the views of teachers and classmates.	CDIO Standard 3

However, Figure 1 also reveals some detailed issues. For example, a notable proportion of students selected uncertain, disagree, or strongly disagree for Question7, suggesting that some students still have doubts about applying the problem-solving methods learned from completing practical tasks to other problem scenarios.

Additionally, Class 1 had a higher percentage of students selecting strongly agree for all 12 questions and a lower percentage selecting uncertain compared to the other two classes. This difference can be attributed to a specific detail in the organization of activities for Class 1. The main presenter for student showcase activities was randomly selected by the group leader through a lottery, meaning that every student in the group had to prepare a presentation and be ready to participate. This approach resulted in higher student engagement during the showcase activities and, consequently, a better sense of accomplishment among students.

During the showcase activities, students also expressed what they had gained from the practical experience. One group said, "Through this teamwork, we learned how to collaborate with others and how to divide labor and work together effectively. We supported and helped each other, solved problems together, and ultimately achieved good results. Teamwork not only improved our work efficiency but also facilitated our learning and growth. In the two practical sessions, we discovered the widespread applications of linear algebra in real-world problems. Through programming practice, I learned how to use linear algebra to solve practical problems such as image processing and data analysis. These practices deepened my understanding of linear algebra and improved our programming abilities."

Another group stated, "The practical experience enhanced our ability to apply linear algebra knowledge to solve real-world problems. During the experiments, each of us encountered difficulties, but we persevered through discussions and ultimately completed the tasks."

The project-driven practical teaching model of linear algebra presented in this paper aims to provide an evaluation basis for teaching objective 2 in the teaching plan. It is an attempt to

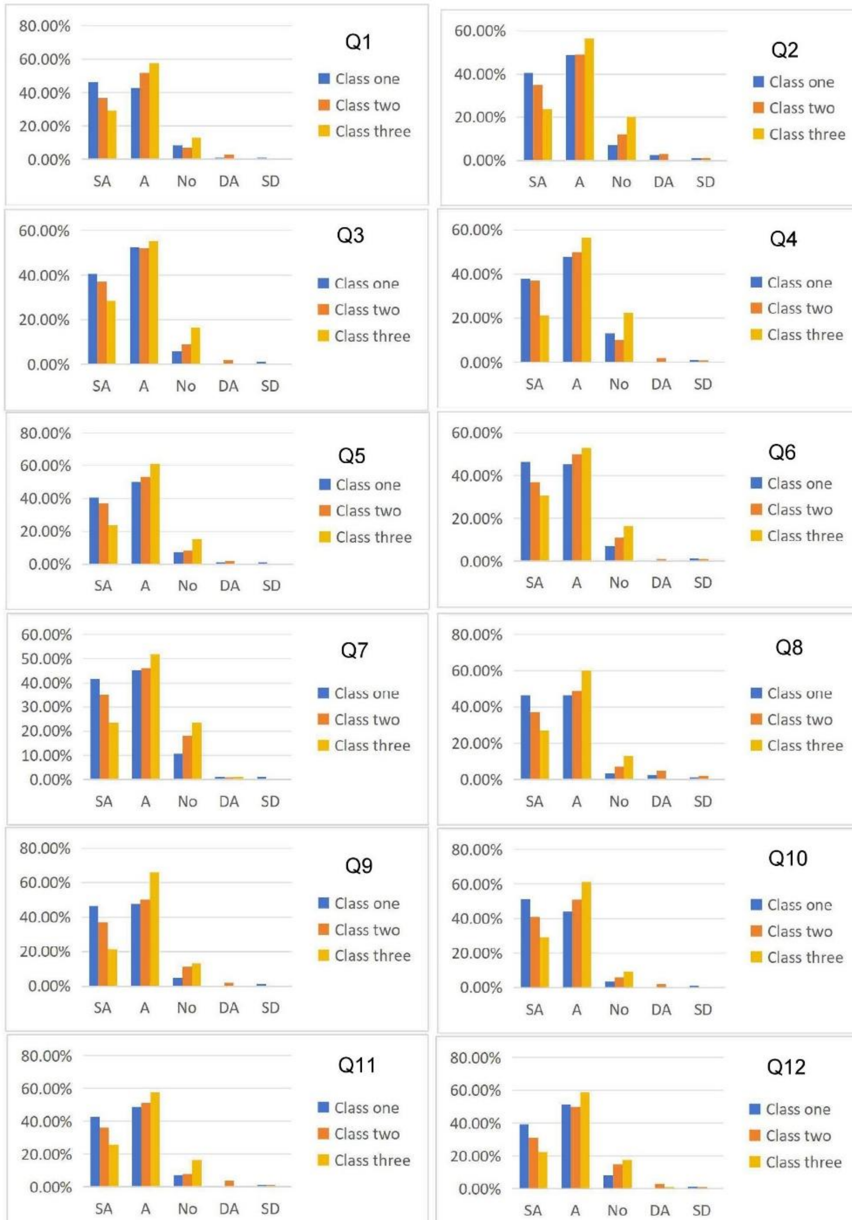

Figure 1 Results of the Student Survey Questionnaire on Project-Based Practical Effectiveness(legend:SA-Strongly agree,A-agree,No-No opinion,DA-Disagree,SD-Strongly disagree)

adopt the CDIO teaching model, limited to a teaching reform experiment within the experimental component of the linear algebra course.Inevitably, some students may not participate enough in project-based practical activities. Therefore, it is suggested that the presenters for the activities could be randomly selected from group members through a lottery, which could improve student engagement. Unfortunately, there was no pre-test survey conducted before the course began, which meant there was no comparison with the data before the teaching implementation.

CONCLUSION

This article successfully applies the CDIO teaching model in the field of engineering education to the university-level linear algebra course. Through the design of practical projects and the assignment of practical tasks, students form project groups and collaborate to complete project tasks within a set time frame. The teacher designs corresponding assessment criteria, and group members present the methods and results used to complete their tasks. Practical grades are obtained through student self-evaluation, inter-group evaluation, and teacher evaluation. This project-driven practical teaching model of linear algebra provides support and evaluation basis for achieving teaching objective 2 of the course. The project design and evaluation criteria correspond to CDIO standards 1, 2, 3, 7, 8, and 10. It is an engineering education model that differs from the traditional linear algebra curriculum, which emphasizes theory over practice and application. It is an successfully implemented engineering education methods to the teaching of mathematical theory courses. In the practice of this project-driven practical teaching model of linear algebra, survey questionnaires and student feedback indicate that students not only deepen their understanding and application of linear algebra concepts but also enhance their practical skills, improve their teamwork spirit and communication abilities, and gain an understanding of the application prospects of linear algebra courses. This lays a foundation for future studies in artificial intelligence-related majors.

FINANCIAL SUPPORT ACKNOWLEDGMENTS

This work was supported by Genetal Program of Beijing Association of Higher Education (NO.MS2023286) and The 2023 Higher Education Science Research Planning Project of the China Association of Higher Education (NO.23SX0411, NO.23SX0424)

REFERENCES

Malmqvist, J., Edström, K., & Rosén, A. (2020). CDIO Standards 3.0 – Updates to the Core CDIO Standards. *In Proceedings of the 16th International CDIO Conference, hosted on-line by Chalmers University of Technology, Gothenburg, Sweden, June 8–11, 2020.*

Zarestky, J., Bigler, M., Brazile, M., Lopes, T. and Bangerth, W., (2022). Reflective Writing Supports Metacognition and Self-regulation in *Graduate Computational Science and Engineering. Computers and Education Open,* 3, p.100085.

Lehmann, M., Christensen, P., Du, X., & Thrane, M. (2008). Problem-oriented and project-based learning (POPBL) as an innovative learning strategy for sustainable development in engineering education. *European Journal of Engineering Education*, 33(3), 283–295.

Bernhard, J., Davidsen, J., & Ryberg, T. (2020). By hand and by computer – a video-ethnographic study of engineering students' representational practices in a design project. In A. Guerra, J. Chen, M. Winther, & A. Kolmos (Eds.), *Educate for the future: PBL, Sustainability and Digitalisation 2020* (pp. 561-570). *Aalborg University Press*.

BIOGRAPHICAL INFORMATION

Jianwei Du has a PhD in applied mathematics and is a professor at Zhiyuan School of Liberal Arts of Beijing Institute of Petrochemical Technology, Beijing, China. Her research interests include Information and signal processing, machine learning and higher education teaching. She has more than 30 years' experience of education.

Shuxia Wang has a PhD in pure mathematics and is an associate professor at Zhiyuan School of Liberal Arts of Beijing Institute of Petrochemical Technology, Beijing, China. Her research interests include nonlinear partial differential equations and higher education teaching.

Qinxian Shi has a master's degree in applied mathematics and is a lecturer at Beijing Institute of Petrochemical Technology. shi is engaged in university mathematics teaching research and practice.

Xinhong Liu is an associate professor in Beijing Institute of Petro-chemical Technology. Her research areas include: nonlife actuarial and statistical models, risk management, and catastrophe data analysis.

Corresponding author

Jianwei Du
Beijing Institute of Petrochemical Technology, Beijing, China.
Zhiyuan School of Liberal Arts
Mathematics Teaching and Research Center
No. 19, Qingyuan North Road, Daxing District, Beijing,102617