ASSESSING STUDENTS' PROFESSIONAL CRITICISM SKILLS – A MATHEMATICS COURSE CASE

Jens Bennedsen

Dept. of Electrical and Computer Engineering, Aarhus University, Denmark

ABSTRACT

Becoming a professional engineer (or other professional careers) requires you to give and receive critique in a non-personal and constructive manner. However, many students cannot provide critique to and/or receive critique from fellow students. In this article, an assessment design for a math course where the student can make and review (critique) a mathematical demonstration (proof) is presented. The students find the assessment method fair but different from what they have seen before. To make room for practising the unusual assessment method, the course uses a flipped-class format. Class time is then used to have the students practice this new form. The practise helps the students to figure out what proper mathematical critique is, and they are more prepared for the final exam as well as constructively critique others' work (not only mathematical work but in general engineering work)

KEYWORDS

Assessment, Math, Communication, Standards: 3, 8, 11

INTRODUCTION

Many different competencies are needed for becoming a professional engineer (or other professional careers). In the CDIO syllabus (CDIO Syllabus 2.0), there are close to 500 topical elements arranged hierarchically. It is not explicitly stated under heading 3 (INTERPERSONAL SKILLS: TEAMWORK AND COMMUNICATION), but addressed by 3.2 Communication that an engineer needs to give and receive critique in a non-personal and constructive manner. However, many students are unable to provide fellow students critique and receive critique from fellow students. In many cases, they take it personally and focus more on "pleasing" the fellow students than giving constructive critique.

Rowntree (1977), in his book "Assessing students: how shall we know them?" notes in the first chapter that "The spirit and style of student assessment defines the de facto curriculum". Ramsden (1992) makes a similar observation: "From our students' point of view, the assessment always defines the actual curriculum" (p. 187) in his influential book "Learning to teach in higher education".

Mathematics is one of the foundations for engineering. Many students see mathematics as a "tool"; something where you find an equation, put in some numbers and get a result. One of my colleagues called the students' way of handling mathematics ”equation hunting” – find the equation that needs the kind of values you have and enter them. Furthermore, students often say that you need to learn the proofs you are doing by heart – if you can memorize them, you will pass (with a good grade). However, mathematics is much more than just equations; if you ask a mathematician (s), he will, in many cases, argue that mathematics is a language. Consequently, we need to be able to ”speak it”. Learning ”to speak” mathematician requires being precise, arguing for the logical transformation of expressions etc.

If being able to professionally critique another piece of work is so important, being able to speak mathematician and what the students' care about is the assessment, we need to find ways to incorporate professional criticism as part of the evaluation. This article describes one example of such an assessment procedure for the final exam: The student must present a given topic (including at least one mathematical proof) orally, and the two other students provide a professional critique of the presentation, including argumentation of why it was good/not so good (using the language). In this case, the grade is determined by 75% of their presentation and 25% of their ability to professionally critique two of their fellow students. The paper includes a detailed description of the rationale for this type of exam, preparing the students in the class for this new type of exam, the exam's execution, and reflection from teachers and students.

RELATED WORK

A lot of different assessment methods have been proposed. Chappuis and Stiggins (2016) list four main categories: selected response (multiple choice questions and short answer questions), essay (poster presentation, written report), performance assessment (case study, practicum, project, and reflective journal/diary), and personal communication (class presentation, interview, and learning contract). Personal communication (interview) includes the (typical) oral examination, and selected-response (short answer question) includes the (standard) written exam. They argue that each category has advantages and disadvantages in assessing different learning outcomes. For example, a multiple-choice test (selected response) can determine all areas of mastery of knowledge but only some kinds of reasoning and arguing.

Written examinations where students solve mathematical problems seems to be the standard way assessment is done (at least in the United Kingdom and the United States, according to Iannone and Simpson (2012)). In their review of assessment practices, Iannone and Simpson (2011) groups assessment into eight categories: *Closed-book examination (CBE), Dissertation, Open-book examination, Multiple choice test, Oral examination, Regular example sheets, Project* and *Presentation*. They find that *"… it is clear then that the assessment diet is relatively restricted; from just under 50% to just under 100% of the final award comes from results students obtain in CBEs (and recall that our method underestimates the proportion of CBE)*" (p. 190). Videnovic (2017) also argues that "*With the increased emphasis on closed book written examinations, the results in this study show that written exams alone are not sufficient to assess students' conceptual knowledge and relational understanding, and therefore, there is a critical need for implementing the oral assessments in mathematics courses.*" (p. 1)

Biggs (1996) notes that "*Tertiary teachers almost universally espouse high level aims for the courses they teach […]. However, generalities such as "To think like a mathematician" or "To become a student-centred teacher, sensitive to individual student's needs" do not imply any particular teaching decisions, which leaves other factors, such as student numbers, or administrative convenience, to determine teaching and assessment methods. The mass* *lecture, and formal examinations, thus continue as the default*" (p. 350). Biggs then suggest that a teacher ask three fundamental questions when designing the assessment of the course:

1. What qualities of learning are we looking for; what performances need to be confirmed in the assessment?
2. Should the assessment be decontextualized or situated?
3. Who should set the criteria for learning, provide the evidence, and assess how well the evidence addresses the objectives?

Ernest (2016) claims that "*conversation, consisting of symbolically mediated exchanges between persons, underpins mathematics, and that it does so in four distinct ways.*

1. *Mathematics is primarily a symbolic activity, using written inscription and language and inevitably addressing a reader, so mathematical knowledge representations are conversational.*
2. *A substantial class of mathematical concepts have conversational structures […]*
3. *The ancient origins and various modern systems of proof are conversational, through dialectic or dialogical reasoning, involving the persuasion of others.*
4. *The epistemological and methodological foundations and acceptance of mathematical knowledge, including the nature and mechanisms of mathematical knowledge genesis and warranting are accounted for by social constructivism through the deployment of conversation in an explicitly and constitutively dialectical way. (p. 205)*

Mathematics is about truth. If there is no direct access to mathematical truth, then access to mathematical truth must be indirect via reason or proof. Reasoning and proving are conversational activities. That implies – if it is a "pure" mathematical course and not a course where you apply mathematical results (i.e., solve a differential equation) – you need to have learning outcomes that focus on "talking math", persuading others by logical arguments etc. From that, you need to design an assessment form that assesses the students' ability to convince others and critically evaluate the arguments. In the following, this paper describes such a course and, in detail, its assessment methods.

If we look at the number of articles presented at different CDIO conferences, teaching mathematics is complex. As an example, Gommer et al. (2016) describe a flipped math course supporting mechanical engineering students. They found that "*After the first weeks, participation during lectures and diagnostic tests dropped dramatically. The pass rate of the course was 66%, compared to 80% in previous years.*" (p. 937). Ferreira et al. (2011) described how they used peer instruction to engage the students more in their math course. They found "*Overall results improved with the PI approach, though mostly at the low end of the scale. PI was thus successful in engaging low-end students to fully participate in the course.*" (p. 7). Cabo and Klaassen (2018) describe a project (PRIME) done at TUDelft to modernize their math courses. They found that their "*findings suggest that the approach taken enhances students' learning performance in maths education. The main results show that students have a more active learning experience compared to the traditional setup of these courses, leading to more engagement, more interaction and better results*." (p. 704)

THE CONTEXT

The case in this article is a discrete math course. The course (ITDMAT) is a 5 ECTS point (60 ECTS = one year of full-time study) course offered to students from five different study programmes: students in the final year of their professional bachelor study in software technology (ICT), electronics (E), power electronics or health technology. Before this course, these students have had two years of mandatory classes and half a year of internship (and some of the students half a year of elective courses). The last study program where students from computer engineering have the class as a mandatory course in their second semester. The students are from at Aarhus University Department of Electronics and Computer Technology. The course is taught in a semester structure (a semester = 14 weeks of teaching and four weeks of examination). This class had 33 students; the course language is English and had two teachers (the author and one more). A more detailed description of a previous course holding can be found in Bennedsen and Lauridsen (2014).

LEARNING OUTCOME

The learning outcome of the course was the following:

When the course is completed, the student is expected to be able to:

- define and analyze the fundamental concepts of propositional logic and predicate logic,
- explain and apply modelling using discrete mathematics such as sets, lists, functions, relations and graphs,
- describe and apply various proof techniques such as induction, contradiction, contraposition, direct.

LEARNING MATERIAL

The course is designed as a flipped classroom course (Bishop & Verleger, 2013). Consequently, the students should acquire the knowledge they typically get from the lectures before the class takes place and use the class time to discuss and get more personalized help. Typically, this is done by some video-mediated material. I have chosen prefabricated videos from different sources in this course and other forms, all freely available on YouTube. None of the videos is produced exclusively for this course but selected by the teachers. Apart from the videos, the assigned textbook (Cusack and Santos (2019)) looks pretty suitable for the purpose since it includes many different exercises that students can work with outside the class hours.

COURSE ARRANGEMENT

The course contains nine topics, each lasting approximately two weeks. The following section describes the assessment method in detail, including seven mandatory written assignments and a final oral exam. There are four class hours per week (two hours Monday and two hours Thursday), so the general structure of the course is like follows:

Monday week X	
1	Highlights of this weeks' topic (this is the lecture)
2	individual work with the mandatory assignment
Thursday week X	
1	workshops[1]
2	workshops (in some weeks trial exam)
Monday week X+1	
1	Feedback on assignment from last round
2	individual work with the mandatory assignment

[1] A workshop is a problem/exercise that students solve in small groups for 15 min and the 5 minutes of presentation of the work by a selected group.

Thursday week X+1	
1	Workshops
2	Preparation and discussion about the exam question related to this week's topic Exam presentation and review

THE ASSESSMENT METHOD

Constructive alignment focuses on ensuring deep learning. The quotes from Ramsden Ramsden (1992) and Rowntree (1977), in the introduction, suggests that we "just" shall assess a way where the students are "forced" to use deeper learning. As Struyven et al. (2005) note, there is no direct correlation between the assessment method and the students' learning approach. They note *"it is obviously easy to induce a surface approach, however, when attempting to induce a deep approach the difficulties seem quite profound*" (p.328). Consequently, one cannot look at the final exam in isolation but need to take a holistic view of the learning.

Mathematics is about conversation; a conversation about *Truth* (Ernest, 2016). Since the conversation (i.e., trying to persuade others that it is true, or – as it is typically called in mathematics – prove it) have special "rules", the students must learn the conversation format. Proofs are different from arguments, as they are comprehended in argumentation theory. Perelman and Olbrechts-Tyteca (1971) separate arguments from demonstrations, such as mathematical proofs. Demonstrations are impersonal; arguments are not. Arguments are, in their entirety, relative to the audience to be influenced. Ernest (2016) describes the following four requirements "*to establish the truth of mathematical knowledge by these means the following conditions are a minimum requirement. We must have:*

1. *A starting set of true axioms or postulates as the foundation for reasoning;*
2. *An agreed set of procedures and rules of proof that preserve truth, with which to derive truths from the axioms;*
3. *A guarantee that the procedures and rules of proof are adequate to establish all the truths of mathematics or at least of the theory in question (completeness); and*
4. *A guarantee that the procedures and rules of proof are safe in warranting only truths of mathematics (consistency)."* (p. 194)

That conversation can be oral or written, so we need to address both kinds of conversation. The assessment method contains both a written and an oral part consequently.

THE WRITTEN ASSESSMENT

Traditional math exercises are written. As described earlier, the students need to persuade (or demonstrate to use the term from Perelman and Olbrechts-Tyteca (1971)) by writing a proof that starts with what has already accepted knowledge, using procedures that preserves the truth, derive at the desired conclusion. Thus, the students need to create a proof and evaluate the proof – is it following the rules, i.e., following the procedures that preserve the truth.

Reading – and memorizing – proofs is at the heart of many mathematics courses. In our case, the students first read proofs (in the material presented before the class) where the procedure steps are described (why is this step preserving the truth). In the material, they also make part of a proof (fill in the details in a proof, see Figure 1)

⋆**Fill in the details 2.4.** Use the definitions of even and odd to prove that the sum of two odd integers is even.

Proof: If x and y are odd, then $x = 2c + 1$ and $y =$ ________ for some integers c and d. Then $x + y = 2c + 1 + 2d + 1 = 2(c + d + 1)$. Now ________ is an integer, so $2(c + d + 1)$ is an ________ integer. □

Figure 1 Fill in the details (Cusack & Santos, 2019, p. 8)

and evaluate proofs (see Figure 2)

⋆**Evaluate 2.8.** Evaluate the following proof that supposedly uses the definition of odd to prove that the product of two odd integers is odd.

Proof: By definition of odd numbers, let a be an odd integer $2n+1$ let b be an odd integer $2a+1$. Then $(2n+1)(2a+1) = 4na + 2n + 1 = 2(2na+1)+1$. Since $2na+1$ is an integer, $2(2na+1)+1$ is an odd integer by definition of odd. □

Evaluation ____

____

____

Figure 2 Evaluation of a "proof" (Cusack & Santos, 2019, p. 10)

In the class hours, the students create proofs on their own (or actually in pairs) called workshops in the course arrangement section.

Seven mandatory assignments do the written part of the assessment of the students. It is not part of their final grade (this is very difficult to do in Denmark due to the regulations on student assessment set by the ministry). Still, to be allowed to sign-up for the final exam, all seven assignments need to be accepted by the teachers. The assignment contains proofs that the students have to make (closely related to the proof they made at the class workshops). The Assessments also follows a *write and constructively critique* pattern. Firstly, the students hand in their solution (i.e., their proofs). They can do that individually or in pairs. After they have handed it in, they have to give feedback to two or three other students' solutions. After that, the teacher evaluates both the solution and the feedback that the students have provided and gives them feedback on both.

ORAL ASSESSMENT

The final exam of the course is oral. Here the students are assessed in groups of three (if the total number of students is not dividable by three, one or two extra students are included, and one or two will be on the bench in each round). During the exam, each student has three roles:

- **Presenter**: The student presents one of the topics. The presentation must include at least one proof and relevant axioms etc. This part takes 12 minutes
- **First reviewer**: The student gives feedback on the presenter. The feedback must focus on the presents ability to make a mathematical demonstration, i.e., their ability to make a valid and sound mathematical presentation
- **Second reviewer:** The students follow up on the first reviewer by commenting on the review and other things that the first reviewer did not address in her/his review

After the first round, the presenter becomes the first reviewer, the first reviewer, the second and the second reviewer becomes the presenter. Then one more round where the students rotate roles. The final grade is determined by 75% of the student's presentation and 25% of the student's review.

The reviewer acts like an opponent. Commonly, opponents are related to argumentation, but in this case, the more have the role as a reviewer of a demonstration of a piece of scientific work. The students have not tried this form of oral assessment before; consequently, we practice during class hours. At the end of two weeks, the students prepare (in groups) a presentation of the topic and do a trial exam. In the beginning, it is my experience that the students are reluctant to critique a fellow student and focus on the presentation technique. Doing this in class has several advantages: 1) The students wrap up the topic by going back and figure out what they would like to include in the presentation (i.e., the most important thing) 2) The students get to know the format and thus are much more relaxed when the final examination comes.

During the Covid-19 pandemic, all teaching has been moved online. This gave rise to the practical problem of "how can a student present proofs online?" Most online teaching tools like Zoom or Teams have a shared whiteboard, but writing on it using a mouse is a problem. Some students have a tablet with a stylus (iPad, touch PC etc.), but that is a minority. Moreover, we do think we can require the students to buy one. Our solution has been using a mobile phone filming a piece of paper – see Figure *3*. If the presentation is done synchronously, the mobile phone is just another participant; if done asynchronously, the students record a video.

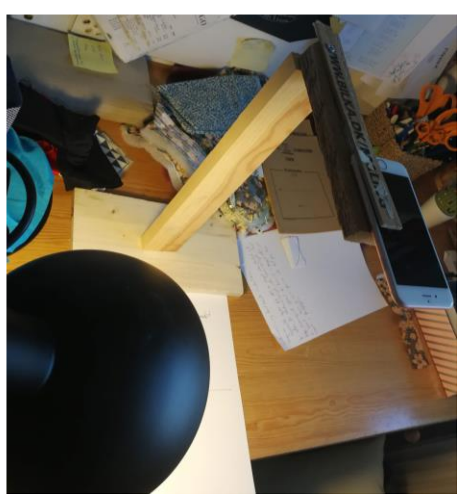

Figure 3 Mobile phone recording a presentation

Using a mobile phone to record the students' presentation made it possible to do that before class. Then – in class – we can put the students in smaller groups (3-4 students per group) and review each other's presentation. The students need more guidance in doing a good review (they typically focus on the presentation technique things rather than the subject things), but in general, this has improved the amount of feedback. Practically, we have gathered the students at the end, where we will watch a presentation together, and the teacher then gives the critique.

THE STUDENTS VOICE

In one of the offerings of this course, we interviewed ten students. The interview focused on the course design, including the creation of the assessment. The interviews were done after the examination. In general, the students found the assessment format fair and supporting the course's goals; there was alignment between the learning outcome and the assessment method. As one of the students said in the interview:

> *"I would say there was a really good connection between the way we were taught and the way the exam was done; it was really well connected."*

A typical problem in Denmark is the so-called "Law of Jante[2]." This implies that you do not want to talk negatively about others etc. Using class time to practice made the students understand that the review did not influence the presenter's grade but the reviewer's grade. As one student explains

> *"It was a really good exam ... It was a bit difficult to find out how much was needed in the individual exam question ... For me, that Law of Jante disappeared a bit when we went in behind that door there ... we knew that ... that we can not make it worse for each other "*

CONCLUSION

In the article, I have presented an assessment design for a math course where the student can make and review (critique) a mathematical demonstration (proof). The students find the assessment method fair but different from what they have seen before. We use a lot of class time to give our rationale for the assessment method and practice this new form. The practise helps the students figure out what proper mathematical critique is, and they are more prepared for the final exam and constructively critique others' work (not only mathematical work but in general engineering work).

Moving the presentation to a digital form has increased the feedback when the students are present in class (either physically or synchronously online). We will continue doing this – also when we are allowed to come back to the university after the Covid-19 pandemic.

REFERENCES

Bennedsen, J., & Lauridsen, A. B. (2014, 22 - 24 February, 2014). An Inverted Math Course. [https://studylib.net/doc/8682660/final-version-of-the-proceedings-here]. Active Learning in Engineering, Caxias do Sul, Brasil. Accessed April 9, 2021

Biggs, J. (1996). Enhancing Teaching through Constructive Alignment. *Higher education*, *32*(3), 347-364.

[2] It characterises not conforming, doing things out of the ordinary, or being personally ambitious as unworthy and inappropriate. See https://en.wikipedia.org/wiki/Law_of_Jante

Bishop, J., & Verleger, M. A. (2013, June 23-26). The Flipped Classroom: A Survey of the Research. 2013 ASEE Annual Conference & Exposition, Atlanta, USA.
Cabo, A., & Klaassen, R. (2018, June 28 - July 2). *ACTIVE LEARNING IN REDESIGNING MATHEMATICS COURSES FOR ENGINEERING STUDENTS* Proceedings of the 14th International CDIO Conference, Kanazawa, Japan.
CDIO Syllabus 2.0. http://cdio.org/benefits-cdio/cdio-syllabus/cdio-syllabus-topical-form. Accessed April 9, 2021
Chappuis, J., & Stiggins, R. (2016). *An Introduction to Student-Involved Assessment FOR Learning* (7th ed.). Pearson.
Cusack , C. A., & Santos, D. A. (2019). *An Active Introduction to Discrete Mathematics and Algorithms* (2.6.4, Maj 2019 ed.). https://cusack.hope.edu/Notes/Notes/Books/Active%20Introduction%20to%20Discrete%20Mathematics%20and%20Algorithms/ActiveIntroToDiscreteMathAndAlgorithms.2.6.4.pdf Accessed April 9, 2021
Ernest, P. (2016). *Mathematics and Values* (B. Larvor, Ed.). Springer International Publishing.
Ferreira, E., Nicola, S., & Figueiredo, I. (2011, June 20-23). *Peer Instruction Method in Introductory Math Courses* The 7th International CDIO Conference, Copenhagen, Denmark.
Gommer, L., Hermsen, E., & Zwier, G. (2016, June 12-16). Flipped Math, Lessons Learned from a Pilot at Mechanical Engineering. Proceedings of the 12th International CDIO Conference, Turku, Finland.
Iannone, P., & Simpson, A. (2011). The summative assessment diet: how we assess in mathematics degrees. *Teaching mathematics and its applications*, *30*(4), 186-196.
Iannone, P., & Simpson, A. (2012). Oral assessment in mathematics: implementation and outcomes. *Teaching mathematics and its applications*, *31*(4), 179-190.
Perelman, C., & Olbrechts-Tyteca, L. (1971). *The new rhetoric : a treatise on argumentation*. University of Notre Dame Press.
Ramsden, P. (1992). *Learning to teach in higher education*. Routledge.
Rowntree, D. (1977). *Assessing students : how shall we know them?* Harper & Row.
Struyven, K., Dochy, F., & Janssens, S. (2005). Students' Perceptions about Evaluation and Assessment in Higher Education: A Review. *Assessment and evaluation in higher education*, *30*(4), 325.
Videnovic, M. (2017). Oral vs. written exams: What are we assessing in Mathematics? *IMVI-Open Mathematical Education Notes*, *7*(1), 1-7.

BIOGRAPHICAL INFORMATION

Dr **Jens Bennedsen** is a Doctor of Philosophy and a Professor in engineering didactics. He received an MSc degree in Computer Science from Aarhus University in 1988 and a Doctor Philosophiae degree in Computer Science from Oslo University in 2007. His research area includes educational methods, technology and curriculum development methodology. He has published more than 80 articles at leading education conferences and in journals. He is a co-leader of the European CDIO region and a former member of the CDIO Council.

Corresponding author

Jens Bennedsen
Department of Electrical and Computer Engineering
Aarhus University
Finlandsgade 22,
DK-8200 Aarhus N
Denmark
jbb@ece.au.dk
+4541893090