UTILIZING GAMIFICATION IN MATHEMATICS COURSES FOR ENGINEERS TO PROMOTE LEARNING

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ABSTRACT

Studies have shown that gamification facilitates students' interest, enjoyment and continuous involvement, allows students to make mistakes anonymously and triggers debates about answers, thus gamification increases students' engagement. In addition, most students respond positively to using online quizzes during the lesson, the main criticism being the short time allowed for each question (Bullón et al., 2018). We examine two aspects of the use of *Kahoot!* online quizzes in three mathematics courses, taught to two groups of first year Israeli engineering students: (1) the effect of using *Kahoot!* quizzes on students' learning outcomes in two questions of the mid-term exams (comparing between traditional teaching and teaching using *Kahoot!* quiz), and (2) students' learning experience. Our findings indicate that in all three courses most of the students did not know how to solve the two questions, neither if the question was taught traditionally nor if it was discussed during the *Kahoot*! lesson. However, in one course (Algebra), students' success was similar in the two groups, which may suggest that using the *Kahoot*! quiz was effective at least as the traditional teaching method. Overall, it is not currently clear whether using the *Kahoot!* quiz has effects on students' achievements. Regarding students' learning experience, our analysis demonstrates that students greatly favor using online quizzes as they feel it deepens their understanding and improves their ability to discuss the taught material. Moreover, the lecturers reported that the use of *Kahoot!* enabled them to review subjects that were taught in the course in an engaging and insightful way, caused a positive change in classroom dynamics and fostered a vivid classroom discussion, addressing issues that otherwise would have remained hidden.

KEYWORDS

Active learning, Gamification, Tertiary Mathematics, Student's engagement, *Kahoot!,* Standards: 2, 8

1. INTRODUCTION

Students' difficulties with learning tertiary level mathematics have been widely reported in the literature and are related to various aspects such as proving (Alcock & Simpson, 2009), the discrepancy between every-day language and formal-logical language (Epp, 2003) and not understanding how to use definitions (Dickerson & Pitman, 2012). Lithner (2011) discusses the tendency of mathematics teachers to reduce mathematical complexity into inefficient rote learning, and attributes part of this tendency to the attempt to cope with the transition from secondary to tertiary mathematics and the relatively low level of student preparation. Indeed, Selden (2005) reports that there are indications of a decrease in students' knowledge and an increase in negative attitudes towards learning mathematics. Other obstacles in the learning of tertiary mathematics are the fast pace and the frequent encounter with new ways of conceptualizing previously well-known concepts which require students to reconstruct former mathematical knowledge. These obstacles, combined with a constitutional pressure to accept more students into studies that require the use of high-level mathematics (such as Engineering) lead to redesigning first-year mathematics courses in a way that might reduce course requirements (Selden, 2005). In order to try and overcome such learning difficulties Lithner (2011) suggests to better adjust tertiary mathematics education to what he refers to as the new societal assignment (mass education), to conduct more research and to form better connections between research and educational development.

A different line of investigation concerns affective aspects of learning tertiary mathematics. The once sharp distinction between the cognitive and affective domains becomes increasingly blurred, and it is assumed that increasing students' positive affect towards learning will improve their achievements and their learning outcomes. Positive affect of students is related to students' motivation and engagement and to the type of lecturer-students communication that prevails in the classroom. One way of increasing students' engagement is gamification, which we next discuss in the framework of active learning.

2. Active learning and gamification in tertiary education

Prince (2004) defines active learning as "any instructional method that engages students in the learning process... requires students to do meaningful learning activities and think about what they are doing" (p. 223). Active learning refers to classroom activities that foster students' engagement, in contrast to traditional lectures where students are mostly passive. Assessing the effectiveness of active learning should address a range of outcomes in addition to factual knowledge, such as student skills (e.g., problem solving) and student attitudes. However, such a broad assessment is difficult, since the success of an active learning approach may be open to interpretation, and many learning outcomes are difficult to measure reliably. A renowned example of a pedagogy that employs active learning in large classes is 'Peer Instruction' (PI) (Crouch & Mazur, 2001). PI requires a consistent use of specially designed classroom tasks, pre-lesson reading materials, appropriate assessment methods and meticulous lesson design. Crouch and Mazur establish the high positive influence of using PI in Physics course on students' achievements, especially regarding conceptual reasoning and quantitative problem solving. Cronhjort et al. (2013) reported that the use of PI in a calculus course for beginning engineering students helped students achieve better results in exams and that students felt motivated and appreciated being more active. However, students also found the method challenging and somewhat frustrating, partly because of difficulties related to the pre-lesson textbook reading. Michael (2006) describes educational case studies in several science courses and concludes that clearly "… there are large bodies of evidence from a number of different fields supporting the effectiveness of active learning" (p. 164). Prince (2004) and Michael (2006) claim that such supporting evidence should encourage faculty to consider nontraditional ways of teaching. Indeed, Peters and Prince (2019) used open-ended, ill-formed problems as means to enhance first-year undergraduate students' higher order thinking skills and complement the competencies students developed through an active learning model.

Gamification is an active learning approach, used to increase students' engagement and motivation in the classroom, and its influence in higher education is constantly growing (Holmes & Gee, 2016; Subhash & Cudney, 2018). According to Holmes and Gee (2016) the recent growing use of games in tertiary education could be an outcome of the attempt to appropriately utilize affordances of new digital technologies to enhance innovative teaching and learning. Subhash and Cudney (2018) offer a vast literature review aimed to identify beneficial components of gamified learning in order to detect contexts in higher education in which gamified learning systems are effectively implemented. They observed that a number of studies demonstrated that gamified learning could be effective in the science studies. Improved student motivation and enjoyment, engagement, and performance were widely reported as the most significant benefits of gamified learning in higher education. Student attitudes (e.g., participation, confidence, interest) were also an important benefit (Bullón et al., 2018; Subhash & Cudney, 2018). The use of achievements and status marks (e.g., bonus points) were among the most frequently used game elements and were identified both as important and suitable for use in higher education. Instant feedback was also an apparently important feature (Subhash & Cudney, 2018). Holmes and Gee (2016) note that nowadays many educators complain about students not being engaged in the lesson, and gamification is a way of “revitalizing education and replacing old, outdated teaching methods with activities and techniques more closely aligned with the expectations and interests of modern, tech-savvy students” (p. 9). The review of Subhash and Cudney (2018) found overwhelming support for other benefits in higher education, such as improved learning outcomes, reduced failure rates and higher average scores (although not necessarily in the final exam scores). Subhash and Cudney recommend further research on gamified learning in engineering disciplines.

In spite of the mentioned benefits, gamification is also criticized by some educators, who perceive gamification as a “re-skinning” of a classroom and not as a real innovation. Others feel that gamification risks implementing play features in meaningless ways, that gamified activities are applied in inappropriate places or that gamification focuses more on “playing” than on deep learning. These critics worry that gamification might cover up for bad course design or bad learning objectives (Holmes & Gee, 2016). Hence one should examine gamification approaches in light of their potential to promote teaching and provide learning opportunities.

For Perdue (2016), the use of games in tertiary level mathematics courses effectively harnesses the time and energy of the students to learn mathematical subjects in enjoyable ways, motivating and engaging them while still allowing them a certain degree of control and autonomy. Perdue tackles the question “How do we want our students to *feel* when they are in class, engaged in solving a math problem, and learning?” (p. 152) and claims that the design of the mathematical task should incorporate a clear quest, a significant reward, choices, unexpectedness and ownership. One way to incorporate such elements in a classroom task is by using a gamification application like *Kahoot!,* which is a game-based online platform designed for social learning. It consists of quizzes of multiple-choice questions and is played by the participants simultaneously and individually via a common screen, web browser and mobile phones, tablets or laptops. When used in classrooms, the teacher prepares quizzes in advance and controls the quiz pace by setting a maximum time for each question. During the quiz all players connect (possibly anonymously) using a generated game PIN. The application does not reveal the right answer and does not continue to the next question until all the students answered a question or time runs out. After each question, *Kahoot!* assigns each student a score that depends both on the answer’s correctness and the time invested in it. After all students answer a question, each student receives personal feedback and the PIN of the current leading student. At the end of the quiz, results can be downloaded by the lecturer (Bullón et al., 2018).

Studies on using online tools have shown advantages such as allowing students to make mistakes anonymously and triggering debates about (in)correct answers. Most students respond positively to the use of online quizzes during the lesson, the main criticism being the short time allowed for each question in the quiz (Bullón et al., 2018). Perdue (2016) reports that she has “…witnessed the power of *Kahoot!* to transform a class of lifeless… students into an excited, energized, motivated group of people competing for mastery of a topic” (p. 153) and recommends the use of *Kahoot!* as a first experience in gamification for lecturers.

Bullón et al. (2018) used *Kahoot!* quizzes in a discrete mathematics course and concluded that most students favor the use of *Kahoot!*, possibly because of the lack of grading. The main students’ criticism related to the competition mode and the time allowed for each question. They further claimed that using *Kahoot!* (or a similar online application) enabled lecturers to trigger student participation, for students were asked (but not forced) to respond. In addition, they claimed that it encourages students to think and reflect. Licorish et al. (2018) interviewed fourteen university students about the influence of *Kahoot!* on classroom dynamics, students' engagement, motivation and learning and found that it: (1) increased students' focus by creating breaks that helped to sustain attention, and (2) provided students opportunities to interact with the lecturer, peers and lecture content in a fun way. In their study, the students noted that the competitive aspect increased their motivation, the anonymity encouraged participation, and that the use of *Kahoot!* increased their knowledge and overall was a valuable and enjoyable learning aid. The last aspect, enjoyment, possibly underlies the former positive aspects. Licorish et al. (2018) remark that due to the small number of participants in their study (14) a larger scale follow-up is recommended.

In this paper, we present findings from a study in which online *Kahoot!* quizzes were used in three mathematics courses taken by first year Engineering students, in order to increase students' engagement, allow lecturers to discuss mathematical concepts in a deep and interesting way and enable the incorporation of high-level questions in the mid-term exam. We present findings concerning students' achievements and learning experience.

3. RESEARCH QUESTIONS

The goal of this research was to study the effect of using *Kahoot!* online quizzes in tertiary mathematics courses taken by first year engineering students. In particular, we wished to learn:

1. The effect of using *Kahoot!* quizzes on students' achievements in the mid-term exams;
2. The effect of using *Kahoot!* quizzes on students' learning experience.

4. METHODOLOGY

4.1 Research setting and population

The study was conducted in three mathematics courses: discrete mathematics, linear algebra and multivariable calculus, taken by first year students in an Engineering college in Israel. Each course was taught by a different lecturer to two different groups of students (20-50 students in each group). In each course, the lecturers used a *Kahoot!* online quiz (50 minutes) during a lecture. The lecturers are all experienced, with over 10 years of experience, teaching mathematics to engineering and mathematics students in universities and colleges in Israel.

4.2 Experimental design: Kahoot! lesson and mid-term exam

We designed a 50 minutes lesson activity of a *Kahoot!* quiz in two courses: discrete mathematics, linear algebra). The activity had two main goals: (1) A mid-term summary of the course; (2) Acquaintance with new high-level questions that were planned to be included in the mid-term exam. Each course was taught to two groups of students. In those courses the mid-term exam included two questions that were taught as follows: in Group 1, Question X was discussed during the *Kahoot!* lesson and Question Y was taught in a traditional frontal lesson, and in Group 2 the opposite. In addition, we performed a short *Kahoot!* activity which included only one question, a different one for each group in a multivariable calculus course. The questions were not taught in a traditional frontal lesson. We describe in details the design of the questions in discrete mathematics course and provide a brief description of the design in the two other courses (for more details see Appendix A).

Discrete Mathematics studies discrete mathematical structures and includes topics from logic, set theory, combinatorics and graph theory. The *Kahoot!* activity included 16 questions. The two questions were (correct answer highlighted in green):

Question X: Let *A* be a set of 5 elements. What is the number of all possible relations above *A*?

a. 25 b. 2^5 c. 2^{25} d. $(2 \cdot 5)^5$

Question Y: Let *R* be an equivalence relation over a set *A*. Then, necessarily there is a function *f: A → R* which is:

a. injective b. surjective c. inverse d. all the answers are correct

Both questions require an understanding of definitions of set theory concepts, connecting definitions of concepts that do not explicitly appear in the question. For example, Question X requires linking the concepts 'relation' and 'a power set' (the latter is not explicitly mentioned). The student must apply the two arguments: a relation above a set *A* is a subset of *A*×*A*; the number of elements in the power set of a set *B* is 2^n, where *n* is the number of elements in *B*.

Linear algebra studies linear systems of equations, vector spaces, and linear transformations. The *Kahoot!* activity included 14 questions. Questions X-Y in this course were aimed to check the students' theoretical background about the connection between a matrix, its properties (e.g. rank) and linear equations. An immediate goal was to check if the students can determine the number of solutions to the corresponding system of linear equations just by observing a matrix, without performing calculations. Other goals were explicating the relation between the rank or determinant of a matrix and the number of solutions. A more general aim was to examine if students would be able to use a specific mathematical tool in more than one way, and to show them it is possible. The multivariable calculus course studies differential, integral and vector calculus for multivariable functions. The *Kahoot!* activity included the study of questions about tangent plane and rate of change of differentiable functions. One of the goals of the activity was to verify if the students master the exact definitions. Furthermore, we were interested in applying the theory presented in the course in an unfamiliar context, trying to determine to what extent engineering students can translate an abstract mathematical concept to a real-world problem.

4.3 Students' Feedback Design

After the *Kahoot!* lessons in discrete mathematics and linear algebra we administered students' feedback questionnaires (based on Bullón et al., 2018) regarding students' learning experience. We consider all answered questionnaires as one set of data, since they do not relate to mathematical content. We present the relevant questions (Q3-Q4):

Q3: Please refer to the following statements and choose the evaluation which best expresses your opinion: *(1) very favorably; (2) somewhat favorably; (3) indifferent; (4) somewhat unfavorably; (5) very unfavorably*.

How do you value the use of:

- Q3.1 technologies (tablets, mobile phones, etc.) for teaching and learning in class?
- Q3.2 *Kahoot!* to develop analytics and decision making related to the taught material?
- Q3.3 *Kahoot!* to expand knowledge and/or deepen understanding of the taught material?
- Q3.4 *Kahoot!* as motivating the relevance of the topic?
- Q3.5 *Kahoot!* as a way to ensure your classroom attendance?
- Q3.6 *Kahoot!* to increase satisfaction with the teaching and learning of the taught material
- Q3.7 *Kahoot!* in lessons in general?

Q4. Please comment on the use of the *Kahoot!* quiz in the lesson (an open question).

Students' answers to Q4 were collected. Repetitive answers were coded according to six themes that emerged from the data (see Table 10 in Section 5.2).

5. FINDINGS

5.1 Students' answers to Kahoot! quizzes and achievements in mid-term exams

5.1.1 Discrete mathematics course

Kahoot! answers

Table 1 presents the overall students' achievements in the *kahoot!* quiz in discrete Mathematics course. Table 2 presents the findings for Questions X-Y. The correct answer is shaded in green.

Table 1. Students' achievements in *Kahoot!* quiz - Discrete mathematics (16 questions)

Discrete Mathematics	Group 1 (N=13 students)	Group 2 (N=12 students)
Total correct answers (%)	35.03%	35.64%

Table 2. Discrete mathematics – Students' answers to Questions X-Y (Allowed time: 60 [sec])

Group 1 (N=13), Question X	(a) 4 (30.7%)	(b) 8 (61.5%)	(c) 0 (0%)	(d) 0 (0%)
Group 2 (N=12), Question Y	(a) 1 (8.3%)	(b) 3 (25%)	(c) 2 (16.7%)	(d) 6 (50%)

Table 2 demonstrates the low success rate of the students in the two groups in solving Questions X and Y. In both classes a detailed explanation of how to solve the questions was given. This exemplifies how the use of the *Kahoot!* activity enabled teaching new implantation and connections of math concepts in an innovative an engaging way.

Mid-term exam achievements

Table 3 summarizes the mid-term exam achievements. The students were considered as successfully answering the question if their question grade was ≥70. The findings indicate that in Group 1 the success rates were significantly higher for the question that was solved in a traditional lesson, whereas in Group 2 there was no difference.

Table 3. The results in the mid-term exam in Discrete mathematics, Questions X and Y

	Group 1	Group 2
Number of students with grade ≥70 - Question X	1 of 18 (Traditional)	2 of 25 (*Kahoot!*)
Number of students with grade ≥70 - Question Y	7 of 18 (*Kahoot!*)	2 of 25 (Traditional)

5.1.2 Linear algebra Course

Kahoot! answers

Table 4 shows the overall achievements in the *Kahoot!* quiz in linear algebra course. Table 5 presents the results for Questions X-Y. The correct answer is shaded in green.

Table 4. Overall Performance in *Kahoot!* quiz for linear algebra (14 questions)

Linear algebra	Group 1 (N=24 students)	Group 2 (N=32 students)
Total correct answers (%)	46.73%	52.9%

Table 5. Linear algebra – Students' answers to Questions X-Y (Allowed time: 120 [sec])

Group 1 (N=24), Question X	(a) 3 (12.5%)	(b) 4 (16.7%)	(c) 8 (33.3%)	(d) 8 (33.3%)
Group 2 (N=32), Question Y	(a) 3 (9.4%)	(b) 5 (15.6%)	(c) 7 (21.9%)	(d) 15 (46.9%)

Mid-term exam achievements

Table 6 presents a summary of the mid-term exam achievements. In both groups, most of the students were not able to get full credit for the two questions. Furthermore, there is no evidence of a significant difference between students' success in each question and in each group when taught via Kahoot! or via a traditional frontal manner.

Table 6. The results in the mid-term exam in linear algebra, Questions X and Y

	Group 1	Group 2
Number of students with grade ≥70 - Question X	1 of 45 (Traditional)	3 of 28 (*Kahoot!*)
Number of students with grade ≥70 - Question Y	4 of 45 (*Kahoot!*)	7 of 28 (Traditional)

5.1.3 Multivariable calculus Course

Kahoot! answers

In this course the *Kahoot!* quiz included only one question, a different question for each group. Table 7 summaries the results for Questions X-Y. The correct answer is shaded in green.

Table 7. Multivariable calculus – Students' answers to Questions X-Y (Allowed time: 90 [sec])

Group 1 (N=26), Question X	(a)13 (50%)	(b) 4 (15.4%)	(c) 7 (26.9%)	(d) 2 (7.7%)
Group 2 (N=21), Question Y	(a) 10 (47.6%)	(b) 6 (28.6%)	(c) 3 (14.3%)	(d) 2 (9.5%)

Mid-term exam achievements

Table 8 presents a summary of the mid-term exam achievements. In both groups only the best students answered the question satisfactorily. This may suggest that students who answered the question correctly in the *Kahoot!* probably answered it correctly in the exam as well.

Table 8. The results in the mid-term exam in multivariable calculus, Questions X and Y

	Group 1 – Question X	Group 2 – Question Y
Number of students with question grade ≥70	8 of 35 (*Kahoot!*)	6 of 43 (*Kahoot!*)

5.2 Students' feedback

The post lesson questionnaire was answered by 51 students. Table 9 presents findings from students' questionnaires. The most frequent answer is shaded. Twenty-four (24) students answered the open question (Q4) about the use of *Kahoot!* quiz in lessons. Table 10 presents our findings, according to the different aspects we defined.

Table 9. Findings from students' questionnaires

	Q3.1	Q3.2	Q3.3	Q3.4	Q3.5	Q3.6	Q3.7
(1) Very favorably	49%	37%	32%	39%	39%	41%	41%
(2) Somewhat favorably	41%	37%	46%	27%	37%	49%	41%
(3) Indifferent	6%	14%	14%	24%	10%	6%	10%
(4) Somewhat unfavorably	4%	10%	6%	6%	6%	0%	6%
(5) Very unfavorably	0%	2%	2%	4%	8%	4%	2%

Table 10. Findings from students' answers to Q4 in the questionnaire

Aspects	Themes that emerged in students' answers
1. General description of the experience	A good way to summarize material, breaks routine, fun, refreshing.
2. Class atmosphere	Creates positive atmosphere in class
3. Insights about the learning process	It motivates learning at class and at home
	It helps prioritizing learning and on which topics to focus
	Lessons learned from mistakes, sharpens a number of inherent emphases on the subject.
4. Students' requests and suggestions	To announce in advance that the next lesson will be a *Kahoot!* quiz and on what subject, so students can prepare in advance.
	To publish a complete and formal solution of the answers.
5. Influence of elements of gamification	Competitiveness improves learning
	The time limitation is stressful
6. Using *Kahoot!* to learn mathematics	Less relevant for practicing proofs and a formal way of writing a full solution

We present two students' quotes that demonstrate aspects 1,2,3,5 in Table 10:

- "In my opinion the use of *Kahoot!* is very refreshing and positive, creates a different and uplifting atmosphere, in addition sharpens a number of understandable emphases on the subject, but with the transparency of the results should be limited, the dimension of time is very influential when solving the exercises and thinking about them".
- "Competitiveness improves, and it's also more fun. In addition, if you come out as bad as I did last time, it only spurred me on to come home and sit down to learn more because I wasn't happy with the outcome. I am in love with the idea of finishing a topic and doing some kind of *Kahoot!* quiz on the topic".

6. DISCUSSION

The findings in Section 5.1 indicate that the use of *Kahoot!* quizzes did not have a positive effect nor a negative effect on students' achievements in the mid-term exam in any of the three courses. Considering the lecturers utilized the *Kahoot!* quizzes not only to summarize and repeat previously taught mathematical content but also as a way to introduce non-routine and advanced questions to the students, and that one of the experimenting lecturers reported that the use of the *Kahoot!* quiz was less time consuming than discussing the same content in a traditional frontal lecture, it seems that teaching via *Kahoot!* is at least as effective as the traditional frontal manner. However, it seems that teaching with *Kahoot!* has other advantages, for example it promotes mathematics teaching that encourages less procedural or algorithmic mathematical thinking, as advised by Lithner (2011). One possible explanation for the lack of positive effect of the *Kahoot!* activity on students' achievements is that the lecturers reported that students treated this activity as "a game", and did not take notes even though important explanations were constantly given by the lecturers throughout the activity. This corresponds to the criticism of Holmes and Gee (2016) that gamification does not necessarily serves learning and that gamification focuses more on "playing the game" than on deep learning. Another possible explanation is that the effect of a single gamified activity is perhaps unable to create a substantial change, as Crouch and Mazur (2001) state: "students often require a period of adjustment to new methods of instruction before their learning improves" (p. 974). So, it is possibly necessary that the lecturer use *Kahoot!* quizzes consistently and thus establish an appropriate classroom culture and norms that will enable the effective use of these quizzes. Thus, we conclude that the use of the *Kahoot!* quiz is indeed an innovative way of triggering deep mathematical discussions in the mathematics classroom in tertiary level, providing that the lecturers who use it succeed in creating an appropriate students' attitude and norms during lessons where these online quizzes are used. This is a matter for future research.

More advantages of using *Kahoot!* quizzes are demonstrated in the findings in Section 5.2. It seems that students clearly favored the use of *Kahoot!* quizzes as a way to develop analytical thinking and decision-making skills, expand their knowledge and deepen their understanding. Moreover, students stated that the use of *Kahoot!* quizzes encouraged them to attend the lesson, increased their motivation and enjoyment and is overall a satisfactory experience. Therefore, we conclude that the use of *Kahoot!* has a positive effect on the learning experience of students, and on their motivation, enjoyment and engagement, in agreement with the current literature. (Licorish et al., 2018; Subhash & Cudney, 2018). However, in contrast to the report of Subhash and Cudney (2018), we did not detect positive effects on students' performances, failure rates or scores. Concerning the "allowed question time", our findings (Table 10) agree with Bullón et al. (2018), that some students find the time limitation in *Kahoot!* Stressful. Yet, our *Kahoot!* data indicate that students' average answering time was less than the time limitation for each question; this phenomenon may be of interest for further investigation (for example, one may investigate if cancelling the time limitations effects students' success).

The different aspects that were raised by the students (Table 10) explain various characteristics of their positive learning experience and offer important insights. One interesting suggestion that was raised by the students is to announce in advance that a *Kahoot!* activity will take place so that students will be able to prepare themselves to the lesson. This suggestion indicates that the students possibly recognize the potential of the *Kahoot!* activity as a learning aid, and wish to make a better use of its offerings. A second interesting student observation is that the *Kahoot!* activity is less relevant for learning proofs and practicing the formal way of writing a full solution to a problem. This observation motivates a future research direction, examining what type of mathematical content, methods and techniques are best supported by the use of *Kahoot!*. As Selden (2005) stated: "The mathematical community should surely keep 'what works'… while responding to new pedagogical challenges and technological tools, and change what doesn't work. So, mathematics education researchers, together with mathematicians, need to ask themselves, what works and why? …[the] technology will change. Whether mathematics professors and teachers will use changes in technology wisely remains to be seen" (p. 144).

Finally, we wish to briefly address the challenge of promoting students' engagement in online courses, which during the Covid-19 pandemic became the prevailing teaching format in tertiary education. Research shows that online instruction is most effective when it requires active student cooperation and utilizes collaborative activities (e.g., group discussions), and when it includes a strong instructor presence, i.e. when the instructor is actively involved in the learning process of the students, preferably in varied ways (Dixon, 2010). Betts (2009) claims that it is important that lecturers integrate effective communication strategies into online course design and instruction if they wish to engage students in learning. However, many students and researchers comment that distance learning courses lack interaction. We suggest that using online platforms such as *Kahoot!* is a convenient and effective way to foster various types of interactions (lecturer-student, student-student and student-content) and group activities, thus engage students in learning during a synchronous online lesson. Since online courses are becoming more and more prevalent, we recommend future research concerning the effects of using *Kahoot!* in online lessons.

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Appendix A

Linear algebra – Questions X-Y

The *Kahoot!* quiz included 14 questions. The correct answers to Questions X-Y are highlighted in green.

Question X: Given that $Adj(A)=\begin{pmatrix}1&0&4\\2&0&8\\7&9&-2\end{pmatrix}$, and denote $A=\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix}$.

For which matrix B, does the equation $Bx=0$, has infinite number of solutions?

a. $B=\begin{pmatrix}a_{21}&a_{23}\\a_{31}&a_{33}\end{pmatrix}$

b. $B=\begin{pmatrix}a_{11}&a_{13}\\a_{31}&a_{33}\end{pmatrix}^T=\begin{pmatrix}a_{11}&a_{31}\\a_{13}&a_{33}\end{pmatrix}$

c. $B=A$

d. none of the above

Question Y: The matrix $\begin{pmatrix}a_{11}&a_{12}&a_{13}&a_{14}&a_{15}\\a_{21}&a_{22}&a_{23}&a_{24}&a_{25}\\a_{31}&a_{32}&a_{33}&a_{34}&a_{35}\end{pmatrix}$ is row equivalent to $\begin{pmatrix}1&2&0&3&0\\0&0&4&-1&0\\0&0&0&0&5\end{pmatrix}$.

For which of the given systems, one does not know how many solutions exist?

a. $\begin{pmatrix}a_{11}&a_{12}&a_{13}&a_{14}\\a_{21}&a_{22}&a_{23}&a_{24}\\a_{31}&a_{32}&a_{33}&a_{34}\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}a_{15}\\a_{25}\\a_{35}\end{pmatrix}$

b $\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}a_{14}\\a_{24}\\a_{34}\end{pmatrix}$

c. $\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}3\\-1\\0\end{pmatrix}$

d. $\begin{pmatrix}a_{11}&a_{13}&a_{15}\\a_{21}&a_{23}&a_{25}\\a_{31}&a_{33}&a_{35}\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$

Multivariable calculus – Questions X-Y

The correct answers to Questions X-Y are highlighted in green.

Question X: The function *f:* $\mathbf{R}^2\rightarrow\mathbf{R}$, *z=f(x,y)* is differentiable. The tangent plane to the graph of *f* at (0, 0) is given by the equation: *6x+8y+z=0*. Find the instantaneous rate of change of *f* in the direction of the unit vector ***u*** = *(0.6, 0.8)*.

a. $(-6)\cdot 0.6+(-8)\cdot 0.8=-1$

b. $|-6|\cdot 6+|-8|\cdot 8=100$

c. $(-8)\cdot 0.6+(-6)\cdot 0.8=-0.96$

d. $0.6:|-6|+0.8:|-8|=0.2$

Question Y: The length of a rectangle is increasing at a rate of 3 [m/sec] while its width is increasing at a rate of 2 [m/sec]. At what rate, in [m^2/sec], is the area of the rectangle changing when its length is 15 meters and its width is 6 meters?

a. 15·3 + 6·2 = 57 [m^2/s]

b. 15+3·2=15+6 =21 [m^2/s]

c. 6·3+15·2=48 [m^2/s]

d. Cannot be answered, there are missing inputs.

Remark: It seems that Question Y is discussed at numerous open online websites such as: Study.com, Quora, Math.stackexchange, Enotes, Socratic.org or Numerade.