REPORT ON ACTIVE CALCULUS COURSE INTRODUCED THE PEER INSTRUCTION

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ABSTRACT

We have introduced the peer instruction well known in physics to coursework in mathematics. The peer instruction is a method to promote mutual learning by students, then it is aimed at essential understandings of learning contents. In the peer instruction in physics, it is an important process to discuss concept tests by students. Through the discussion, they notice differences between the exact meaning of physical contents and the intuitively understandings in real life. The learning contents in calculus course also include various aspects, such as exact definition, concept in calculation, geometric significance, sense in application, historical background, and so on. Japanese students often learn only one face of learning contents, usually how to solve problems. By showing the concept tests about learning contents, the students notice the various aspects of what they have used for solving problems. It is noteworthy that this will happen in the discussions between students, that is, peer instruction. In this conference, we report on class case introducing the peer instruction in calculus course. Indeed, such a class is active despite the mathematics class, discussions will begin as soon as the display of the concept test. Thus, it became a calculus course for training engineers, near CDIO principles. We have practiced over the course of a year and a half, revealed the difficult points and improved them. Then, we report on difficult and improving points in mathematics class and verification on whether they understand various aspects of mathematical contents.

KEYWORDS

Peer instruction, Mathematical concept test, Group works in mathematics classes and CDIO Standards: 7, 8, 10

INTRODUCTION

We have introduced the peer instruction (PI) well known in physics (Mazur, 1997) to coursework in mathematics. This method is a lesson management method to promote learning among students. It is a method to deepen the understanding of the concept of learning matters by discussing among "students" the "rationale for the validity of the correct answer" to the concept problem. Dialogue scenes with others are important in the PI method. In particular, interact with others of equal relationship is, to enhance the learning effect is pointed out in the like authors of the previous study (Yamaoka, et. al., 2015). Based on previous studies, we started calculus classes introducing PI in 2016. We applied it to the class of the first year student in our college, i.e., Kanazawa Institute of Technology (KIT). The target subject is “Integrated Math and Science for Engineering” where students learn calculus and basic dynamics and their application to engineering.

In mathematics classes, the peer instruction method has been used, for example, Pilzer, S. (2001) and Lucas, A. (2009). However, our research group tried it independently, and later learned that precedent case. Our originality is to classify concept problems used for peer instruction and to present the effective ordering sequence. While we are currently examining the effect, we can see effects such as improvement of concept understanding and reduction of typical misunderstanding answer (Yamaoka, et. al., 2017). By arranging the order to be quizzed, it becomes a lesson that thinks even more advanced problems, e.g. application of calculus to physical and engineering problems and reconsidering mathematics concepts and formulas. Thus, students have experienced integrated learning and the class have been activated.

As we have seen in previous studies (Ferreira, et. al. 2011 and 2014, Cronhjort, M., et. al. 2013), the peer instruction method sufficiently mentions CDIO Implementation. In particular, our practice satisfies CDIO standards 7 (Integrated Learning Experiences), 8 (Active Learning), and 10 (Enhancement of Faculty Teaching Competence).

In the following, we will describe practical methods in class and how to implement mathematical concept problems.

CLASS DESIGN

We recommend learning outside of class hours to ensure group work hours during class. What to keep in mind when doing this is to confirm the basic matters and practice as homework and to treat evolving topics with group work.

Lesson Framework

Our lesson consists of the following four elements,

1) Quiz for checking preliminary exercise,
2) Lecture on mathematical concept and formula,
3) Group work for improving understanding of learning matters
4) Explanation of preliminary exercise.

The lesson time is 90 minutes, and the time allocation is as shown in figure 1.

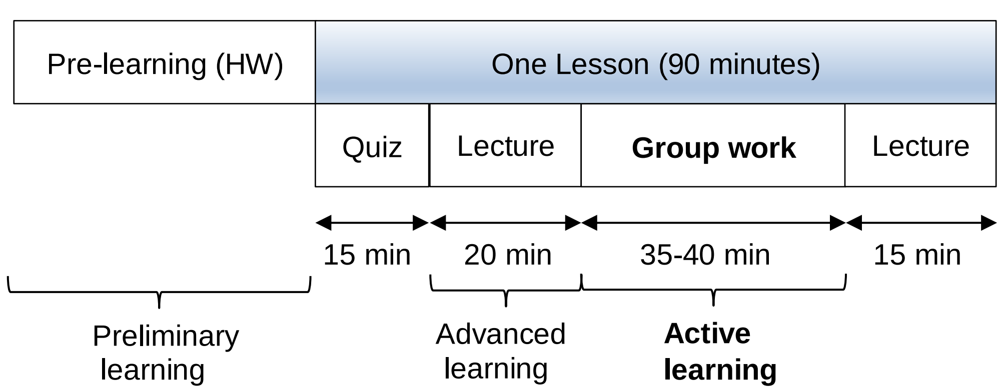


Figure 1. Lesson framework

Quiz for Checking Preliminary Exercise

By referring to the explanation at the end of the previous lesson, students can solve simple exercises and learn basic matters. We check the degree of comprehension for the preliminary exercise with a quiz of about 10 minutes at the beginning of each lesson. Then, they receive the answer after the quiz, immediately confirm their comprehension, and face the lesson of the day.

Lecture on Mathematical Concept and Formula

In this part, we will explain mathematical concepts to be understood through preliminary exercise, and introduce application examples of basic matters and developmental formulas. Thus, we make our students aware of various aspects of what they used to solve problems. Moreover, we can finish the developmental content in the first half of the lesson and it was possible to secure the time for the next group learning.

Group Work for Improving Understanding of Learning Matters

This part is important in our lesson, and in this part teachers provide topics for students to actively learn. For more information on group work, although described in the following subsections, teachers incorporate the scene of some dialogue in this part. Incorporating the scene of dialogue in each class, even an inexperienced students to dialogue at the beginning semester, going so as to participate actively in this group work. As a result, it is also possible to incorporate various forms of group learning.

Explanation of Preliminary Exercise

In the last part of the lesson, teachers explain the things necessary for homework until next time and finish the lesson. The only thing to watch out here is the introduction of basic matters and review of the basic formula. In order for students to learn by themselves, it is necessary to pay attention so that teachers do not teach too much. As a result, it is possible to foster even autonomous learning attitude of students

Group Work

The feature of the classroom is that students have many colleagues and we have prepared efforts to make the most of it. In the most important group work in the class, we adopted the following activities:

- Review and development of quiz for checking preliminary exercise,
- Mutual learning of various aspects of mathematical concept by students,
- Discussion on mathematical concept problems.

If the teacher explains various aspects of mathematical concept, students will not catch up with all aspects unless it is according to their understanding level, or only understanding of the emphasized aspect. Group work by students is the most suitable for students to progress according to their understanding level. In group work, students with poor understanding have the potential to fill up the difference at a stroke by dialogue with well-understood students. Furthermore, for more advanced and applied topics, students will be able to propose various ideas. Teachers need to be a facilitator in the class so that the group work of the students becomes active.

Review and development of Quiz for Checking Preliminary Exercise

Following lectures before group work, let students work on application examples. The task is to prepare some applications that avoids calculating as much as possible and discusses the significance and advantages of mathematically thinking in groups. For example, after definition of the Riemann sum, we introduced numerical calculations such as trapezoidal formula and Simpson method (as shown in figure 2), and discussed how to improve the accuracy of approximation.

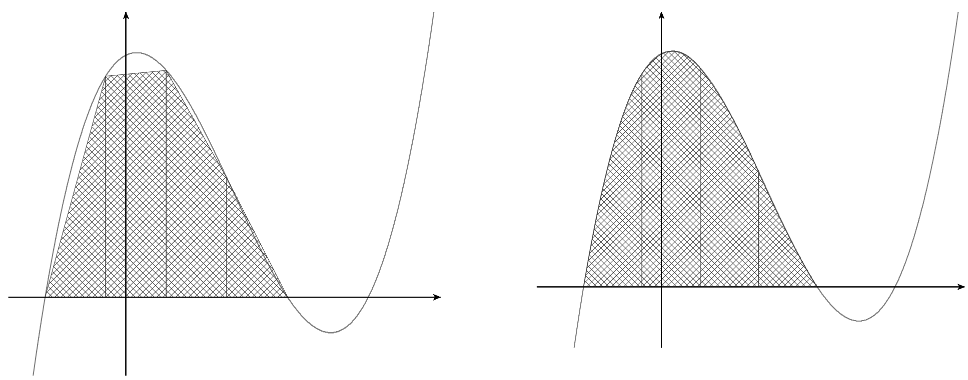
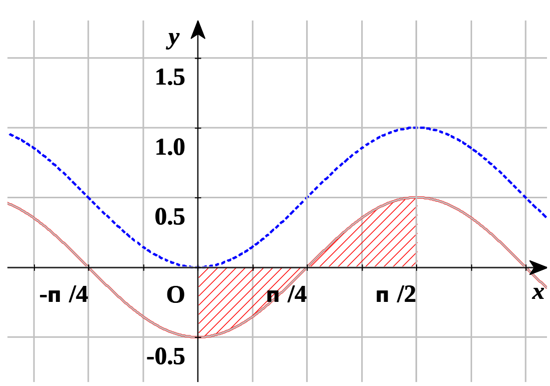


Figure 2. Trapezoidal formulas (left) and Simpson methods (right)

The trapezoidal formula is a contrivance to improve the accuracy of the shape approximated, the Simpson method use the fundamental theorem of calculus. Through group work for the Riemann sum defining the definite integral, the students are guided to the integral calculation by the fundamental theorem of calculus.

Mutual Learning by Students

Following lectures before group work, let students exercise developmental formulas. At this time, giving derivation of the formulas as the task, the teaching in the group becomes active. Often in mathematics, the understanding progresses more by derivation of the formulas, so the activities in the group appear effectively. Once the understanding of formulas is deepened, exercises can be easily done by individuals. For example, after calculating the definite integral using trigonometric half-angle formulas, let students consider the area in the graph as shown in figure 3.



$$\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$$
$$= \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\,dx$$
$$= \int_0^{\frac{\pi}{2}} \frac{1}{2}\,dx + \int_0^{\frac{\pi}{2}} \frac{-\cos 2x}{2}\,dx$$
$$= \frac{\pi}{4} + 0$$

Figure 3. Comparison of areas of figures drawn by trigonometric functions

The problem is also an introduction to consider that the value of the definite integral for each cycle is vanishes and that the values of the definite integral for different integration interval, and an introduction to the Wallis formula.

$$\int_0^{\frac{\pi}{2}} \sin^n x\, dx = \begin{cases} \dfrac{n-1}{n} \cdot \dfrac{n-3}{n-2} \cdot \dfrac{n-5}{n-4} \cdot \cdots \cdot \dfrac{2}{3} \cdot 1 & (n : \mathrm{odd}) \\ \dfrac{n-1}{n} \cdot \dfrac{n-3}{n-2} \cdot \dfrac{n-5}{n-4} \cdot \cdots \cdot \dfrac{1}{2} \cdot \dfrac{\pi}{2} & (n : \mathrm{even}) \end{cases}$$

As a result of incorporating mutual learning by students into the class, the correct answer rate of the exam questions using the Wallis formula was 53% in FY 2016, up to 77% in FY 2017. Both classes cover the same department, and the number of classes is also comparable.
For the derivation of Wallis' formula, the recurrence formula using partial integration is used and understanding of the area integration tends to be lost. When n = 2, it can be combined with trigonometric half-angle formula, and its understanding also takes on the aspect of the area. Such heuristic understanding is born in teaching between students. Moreover, the students deepen their understanding of formulas.

Discussion on Mathematical Concept Problems

In some cases, we took advantage of mathematical concept problems to make the developmental formulas more deeply. Alternatively, I presented a conceptual problem to aim for a deep understanding of the basic formula for introducing a developmental formula. In this case, we exchange time with the lecture of the first half, and immediately after the check we have begun a conceptual problem. To raise sensory understanding, the conceptual problem is suitable, and we presented graph like figure 3 before presenting the conceptual question.

MATHEMATICAL CONCEPT PROBLEMS

The background of inventing the group work as in the previous section is in a previous study that began to classify the characteristics of mathematical concept problems (Yamaoka, et. al. 2017). As introduced in the research (Ferreira, et. al. 2011), the conceptual problem is presented twice in the peer instruction, and the improvement of the correct answer rate by the discussion is examined. We found that there are differences in improving the correct answer rate depending on the characteristics of the conceptual problem. In addition, when consulting related problems, we found an ideal improvement in the correct answer rate than when presented independently. In this section, we describe the characteristics of mathematical concept problems.

Classification of Mathematical Concept Problems

The conceptual problem as the subject of discussion is preferably to logically combine learning matters. Such conceptual problems, we have classified it as,
- Knowledge-based problem combining various concepts,
- Knowledge application type problem that sees a concept from another viewpoint.

The former knowledge utilization type problem plays a role of deepening "inquiry mind" in mathematical learning (for example, the problem on the left of the figure 4), the latter knowledge applied type problem is to connect between subjects and fields to improve "advanced learning motivation" (for example, the problem on the right of the figure 4).

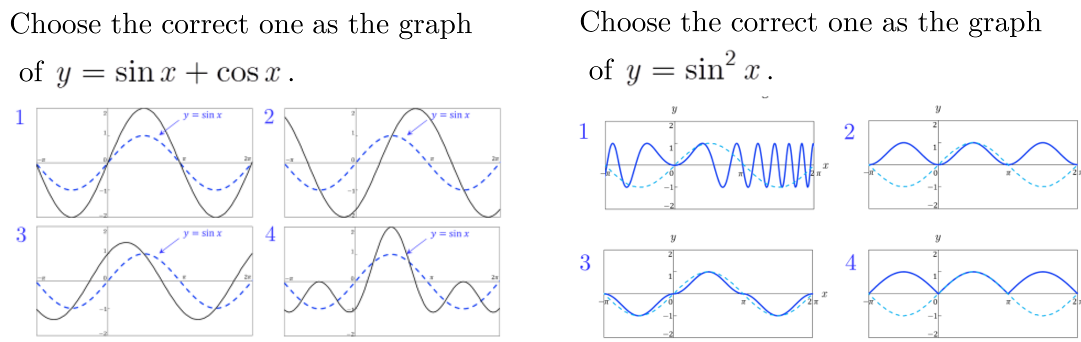


Figure 4. Examples of mathematical concept problems

In the above example, the right answer rate on the left rose from 68.4% to 98.3% (gain 0.95) and the right correct answer rate rose from 51.7% to 83.1% (gain 0.65), where the gain defined to be $(r_f - r_i)/(100 - r_i)$ with the right answer rate r_i of the class at the first answer and the right answer rate r_f of the class at the second answer. The gain of former knowledge utilization type problem tends to be good, and the gain of latter knowledge applied type problem tends to get worse. Understanding the differences between these types, conceptual problems should be used effectively in peer instructions. By presenting the basic confirmation problem, we can improve understanding of the latter type. In order to be effective questions, the classification of the concept problem is an important research.

Implementation of Mathematical Concept Problems

In the case of presenting the conceptual problem of the degree of difficulty conforming to the proficiency level of the class, the student who answers the correct answer at the first answer is 60 to 70% and the student who answers the correct answer at the second answer is 80 to 90%. These cases is considered that discussion between students is effectively done.
On the other hand, when presenting an easy conceptual problem to the degree of familiarity of the class, 90% of the students answered the correct answer at the first answer and tended to infer from the result and answer the correct answer at the second time. However, even with such a problem can be utilized as

- Confirmation of already studied concept,
- Tips on issues to be presented next.

This is because we wanted to prepare knowledge materials necessary for discussion among students in various ways in the classroom. As a method of presenting other discussion materials as follows:

- Confirmation of the preparatory test and its commentary: remain as hand data,
- Explanation of learning matters by writing on blackboard: remaining as materials beside the screen,
- Sequence of problems that makes you think consecutively: Remain as last memories.

If we do not prepare such hints, the discussion by students will not be activated, so faculty by teachers is also important. At that time, while considering not to induce deliberate discussion, we must conduct a voice call and a desk patrol for class revitalization.

CONCLUDING REMARKS

The mathematical concept problem presented in the peer instruction should be examined after clarifying the path of acquisition of mathematical concepts. We believe that it is also the future task of our research group to have a systematic overview of mathematical concepts. Self-affirmation in group work is also an important factor for improving academic ability. In order to realize this, it is important to realize that discussion and understanding has deepened. It is a future work to seek out scenes that make people feel self-affirmation.

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BIOGRAPHICAL INFORMATION

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