"COMMUNICATION OF MATHEMATICS" AS A TOOL TO IMPROVE STUDENTS' GENERAL COMMUNICATIVE SKILLS

Jan-Åke Larsson[†]
Department of Mathematics
Linköping University
SE-582 16 Linköping
Sweden

Abstract

This is an account of an attempt to improve students' communicative skills, with a focus on mathematics. The intent is to give the students skill and experience in communicating in an environment where precision is important, both in mathematics and science in general, but also in engineering. The first part of the course is intended to improve the students' ability to follow a logical argument, especially long (even infinite) chains of logical arguments. Later parts of the course focus more on the practice of presentation of, discussion of, and writing mathematics. Examination is not by a written exam, the examination consists of students' participation in oral presentations and the ensuing discussions, a one-page handwritten hand-in at the start of the course, and finally a short typed piece on a suitable mathematical topic. Experiences from this first attempt are discussed, and the most striking effect is the visibly improving oral communication skills of the students as the course proceeds. There are also indications that participation in this course is beneficial to later mathematics courses, but only for the able students. We do expect an improved overall performance of the students but there is no clear effect as yet, partly because there has not passed enough (read "any") time after the finished course, but perhaps also because the sample is small.

Keywords: Communication of Mathematics, Communications

Introduction

We have seen a decline in student mathematical knowledge and skills in the first year of the engineering program "Applied Physics and Electrical Engineering" in Linköping for quite some time. One could speculate at length on, and investigate at depth into the severity of this and the

† jalar@mai.liu.se

reasons for it, but that is not the subject of the present paper. Instead we will turn to the possible effects, and remedies. One effect is the decrease in student performance in the first year, and we are addressing that problem as best as we can. But here, we will have a look at another problem: the performance and stimulation of the more capable students.

The mathematics teaching in the first year is hampered by the substantial number of struggling students, and even if the curriculum has not changed, there is an effect of the changed composition of the class. The more advanced questions do not arise so often in class, and more advanced topics related to the current subject of study will not get discussed; at least, not so often as they used to be. The standard solution would be to provide an advanced course or an honors course in mathematics to these students, targeting the top mathematics students. As one would guess, such possibilities already exist within the program, and do engage some good mathematics students.

An alternative is to use components of the CDIO syllabus used in this engineering program, specifically the *communications* topic (topic 3.2 in the current CDIO syllabus), and use it as a basis for a voluntary course on *Communication of Mathematics.* It is to be expected that a course like this would be of interest not only to the top mathematics students, but also to good students which are not necessarily the target of the honors courses mentioned above.

The general idea is that the course should expose the students to more mathematics, improve their ability to follow and/or present a logical argument, and to give them some experience in oral and written communication. And of course, we expect that this will improve their results, certainly in other mathematics courses, but also in general, in their other subjects as well. We also would not be surprised if there is a lasting effect, reaching into their professional careers, but this is admittedly difficult to check.

The communication part of the course follows the communications topic in the CDIO syllabus to a large extent, and the mathematics content is chosen to

a) improve logical reasoning skills and explain the formal structure of mathematics,

b) augment the parallel courses with more advanced results than treated there, and

c) introduce the students to parts of mathematics not shown elsewhere to raise their interest in the subject as such.

Course description

Throughout the course, the students have been split into small groups to create an atmosphere where all participants could take part in the discussion. The aim was to have 8-10 students in each group. In this, the first year, twenty-four students chose to follow this course. This was somewhat more than expected, but even so, only two groups were created.

The course consisted of three parts; logic and structure, oral, and written communication. It started with a meeting where we got to know each other and discussed on a general level our different

backgrounds, our expectations, and our previous experience from communication. We also talked about good and bad examples from our experience. Quite naturally, I (being the teacher) drew from this discussion to indicate the topics I'd include and work from that.

The following meetings were about mathematical logic, a rather dry topic from the point of view of oral presentation, but nevertheless important in the basis of mathematical reasoning. The material was taken from one of our standard logic texts (Asratian, Björn, and Turesson, 2006). Within mathematics, the subject does also relate to the "Communications Structure" subtopic of the CDIO syllabus. I gave a few 45 minute lectures on this topic while commenting on my style of presentation, pointing out deliberate *and* accidental mistakes that I did as we moved on. These lectures were interspersed with a larger number of short (5 minute) presentations by the students on exercises in mathematical logic, calculus and linear algebra.

Continuing on from there, I talked about implication and equivalence, what makes a mathematical theorem, the structure of a proof, and some proof techniques. I also talked about presentation technique, objectives, audience, and style, using Alley (2003) as a starting point. This basically covered the "Communications strategy" and the "Oral communication and interpersonal communications" subtopics in the CDIO syllabus.

Here, the students were given topics for a longer (15 minute) presentation, on more advanced mathematics. Some students were given theorems and proofs from their parallel courses, calculus and linear algebra, using their normal course texts (Forsling and Neymark, 2004; Janfalk 2007). Some were given more advanced results, normally discussed in honors courses (e.g., from Rudin, 1976), and some were given other results not normally in any of our courses, from Aigner and Ziegler (2004). In each case I did my best to adjust the difficulty to what I knew about the student. Needless to say, I sometimes got it right, on occasion gave a too easy task, and more than once gave a too difficult one. Both the short and the long presentations were followed by discussion, initially on the subject in question and followed by comments on the presentation. In the cases where the task was obviously too difficult, I helped fill in details and then went on to discuss the merits of the presentation as such.

It should be mentioned that much of the material that I was intending to go through was covered to just by keeping the discussion afloat, and in a few cases, filling out at the end. I also drew from my own experience as a teacher and as a speaker, trying to relate to various audiences and circumstances. This was sufficient to give the students ample hints on what to do and what not to do, without resorting to lecturing on it. I found that I often could use my notes to simply repeat and perhaps bring structure to what we just had found together, in practice.

I did give more of a lecture on the structure of a computer-projector presentation (as in "Electronic/Multimedia Communication"), because we did not have the time in this course to make a student exercise of it. The lecture was based on advice given in Alley (2003). The most vivid practical example was that of Murphy's law, the projector broke while I was giving the talk in one group, to the general amusement of the students.

The next topic was "Written Communication." We covered with much of the same kind of advice for mathematical, scientific, and engineering texts as we earlier had covered for oral presentations, but this time more in the form of lectures, although there were some discussion also. Some attention was given to the general structure of a written scientific paper, both short and long, and discussed cross-referencing and citations. I also gave some more specific advice on how to write *mathematical* text and how to join mathematics to ordinary text, much of the advice I gave can be found in Steenrod et al (1971), Gillman (1987), and Knuth, Larrabee and Roberts (1989).

We had one meeting in one of our computer laboratories, and spent some time learning the LaTeX typesetting system. For the reader not familiar with this system, it perhaps suffices to say that it is a typesetting system widely used in mathematics, and in science in general. Unfortunately it also has a steep learning curve. While the students are allowed to produce their second *typed* hand-in in OpenOffice or MS Word, LaTeX is preferred.

One aspect of communication not mentioned in the syllabus is how to criticize presentations, both written and oral, and how to do this in an effective manner. We spent some time reading and criticizing mathematical entries on Swedish Wikipedia. I chose Wikipedia because it seemed difficult to get the students to criticize each other in a good way. This proved to be fruitful; we had a good discussion and both bad and good entries exist, although the overall quality was lower than I had expected.

At the end of the course I decided to give the students a real-life example from my own work in Quantum Cryptography. I talked about a certain paper I had presented at two conferences Sept 2006 and March 2007 (no proceedings) and as a journal paper (Cederlöf and Larsson, 2006). The first was a 30-minute presentation and the second was a 12-minute presentation, both using electronic media. The paper is eight pages long, formatted for a major specialist journal in the subject area. We discussed at some length, first the content, and then what to consider when writing the paper; constructing a long presentation; and constructing a short presentation.

The course is not yet finished as of this writing, but the very last meetings will be devoted to graphics ("Graphical communication" in the CDIO syllabus), style in written text (from Strunk and White, 2000), feedback on the typed hand-in to the students, and feedback on the course as a whole to me. I have got some preliminary reactions and these will be discussed below.

Experiences

As mentioned, the course is not quite finished at the time of writing, but I have tried to estimate the effect of following the course on results of other mathematical courses; see Fig 1. The figure shows results before the course versus results after. The trend is clear: good students do better after the communication course while less able students do worse. Both effects might have been expected, the latter because the students will spend time on the communication course, quite possibly more time than they really have to spare. It should be noted that the effect is not large enough to be statistically significant, but that is not to be expected after only one run of the course.

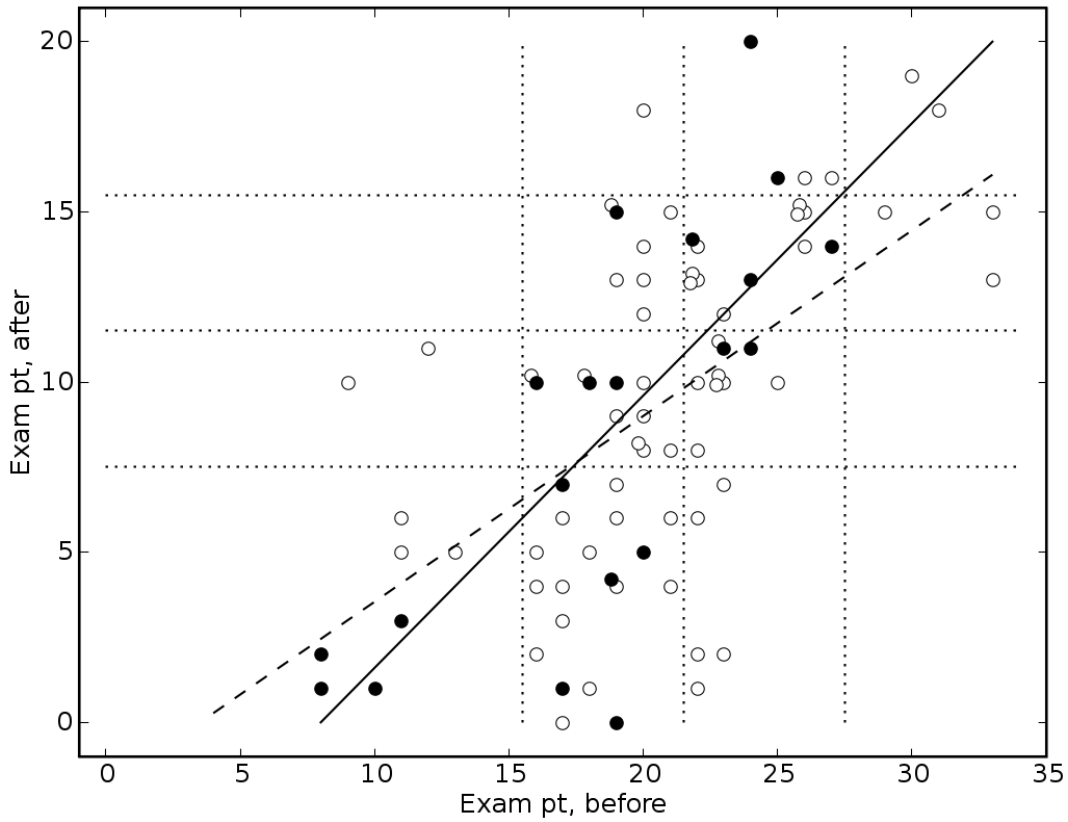

Fig 1: Examination points from a course examined before the communication course ("Mathematics, basic course"), plotted against the points from another course that finished late in the communications course ("One-variable calculus, II"). The black dots are students who have taken the communications course; the white dots are students that have not. The grades are divided by the dotted lines and are "Failed," "3," "4," and "5," from left to right and from bottom to top, respectively. One immediate observation is that the top students on the first course have not chosen the communications course. Another is that, while there is no linear dependence, the trend is obvious (solid line: communication students, dashed line: others), the communications course is beneficial for students with a high result on the first exam, while it is the opposite for students with a low result.

The most obvious effect *in class* was the visibly improving oral presentation skills of the students. It is evident that practice in presentations and more systematic use of reasoning about and drawing experience from other peoples presentations improve the student skills considerably, at least for the relatively untrained students of this year's course. These oral presentations do also seem to be the most appreciated part of the course, possibly because the students clearly notice their own progress.

Another experience is that group discussions are difficult in this setting; there is little tradition to engage in discussions in class in this particular engineering program. The reasons for this are difficult to track, but suggested reasons include the particular student type attracted by the

program, and/or attitudes inherited during the interaction with the older students in the first weeks of study. Another much simpler explanation is shyness and uncertainty of ones own capabilities, but also in this instance one reason may be the somewhat large groups that resulted from the larger-than-expected interest. It is possible to maintain a decent level of student interaction in the discussions, but it takes some effort and afterthought from the teacher.

Conclusion

There are some things that need to be amended to the next instance of the course. These include but are not limited to:

a) The CDIO syllabus subtopic "Graphical communication" has only briefly been touched in this first instance of the course. Next year, there will be more on this topic.

b) A better grip on the student's previous results, to help in choosing the appropriate difficulty in tasks given to the students. It is my belief that students are most helped by being given tasks only slightly above their previous achievements.

c) To work more with written communication. The general feeling of the students and of myself is that most of the discussion has been centered around oral communication. It is possible that this feeling is caused by the order in which the material was presented, but I will take some care to make the emphasis more even next year.

Another change is the written hand-in. Next year, the hand-in will be via the net. The poor quality of the mathematics section of Swedish Wikipedia appalled me. Next year's students will be given mathematics topics suitable for writing a short Wikipedia article. The list of needing subjects is virtually endless, so there will be much to choose from in years to come.

The course certainly seems to have been a success. The students seem satisfied, and naturally have opinions on the content and how it should change to make the course better. The visible improvement in oral presentation skills and the decreased nervousness of the students are both positive signs. There is a noticeable effect on the results in mathematics courses, but it remains to be seen what the long-term effects are, and also what the effects are in other subjects.

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